Single Driven Vortex Chain in a Confining Potential

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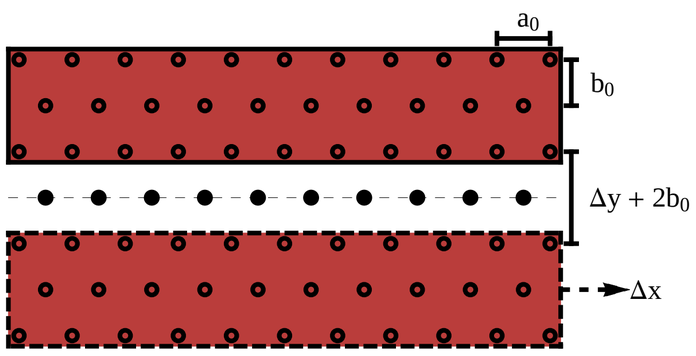
Abstract

We numerically study thin flow channels of superconducting vortex matter in a periodic confining potential caused by pinned vortices on the channel edges. The channel width is varied in order to study the transition between single and double (bifurcated) chain configurations of flowing vortices. At very low temperatures, thermal fluctuations are sufficiently weak enough to maintain the single chain configuration for long time scales above the critical channel width. The order parameter shows hysteresis in the single-double-single transition path if the chain doesn't thermalise quickly enough. For the undriven system, we are able to examine the energetics of the mechanism to understand why the single chain appears metastable.

Thin Channels



- Pinned and flowing vortices are represented by empty and filled circles, respectively. The additional **width of the channel** is controlled by Δy, and the lower **channel off-set** is controlled by Δx. The central dashed line is defined as $\mathbf{y = 0}$. We define $\mathbf{N}$ to be the **number of vortices** to fill each unit cell once, and only once, in the diagram above. Additional **interstitial vortices** are counted using $\mathbf{n}$. In the Abrikosov lattice, $\mathbf{b_0 = a_0\sqrt{3}/2}$.

- We can build similar soft matter systems using other types of particles, for example: Wigner crystals [1], colloidal suspensions, and hard disks [2].

- Other repulsive potentials can generate similar systems, e.g. $\exp\left(-r^2\right)$, $1/r$, or $\exp\left(-r\right)/r$.

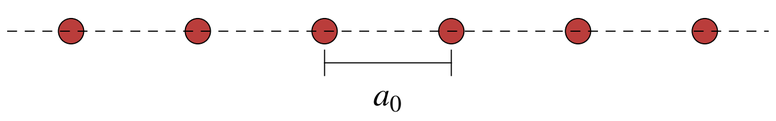
Order Parameter

- The amplitude parameter characterises the single and bifurcated state.

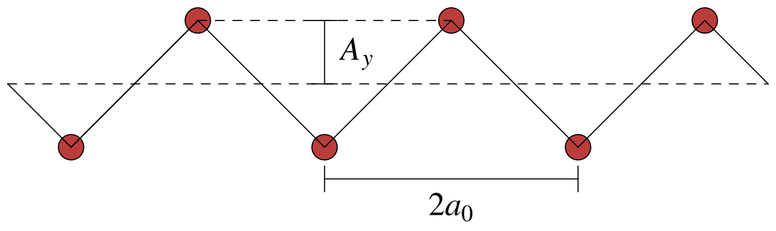
$$A_y = \sqrt{\frac{\sum_i^{N+n} y_i^2}{N+n}}$$

- For the single chain, each $y_i = 0$, so $A_y = 0$. For the bifurcated chain, $A_y > 0$.

Chain Structures



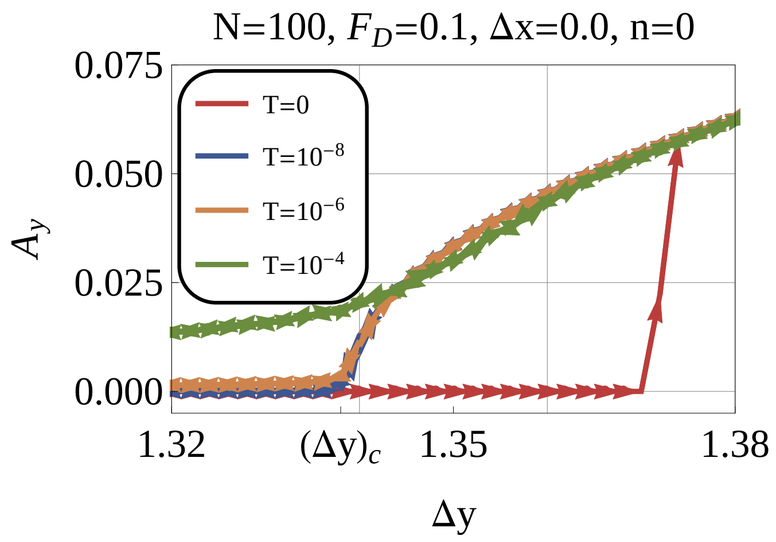
- Single chains are 1D for commensurate values of Δx.



- For $\Delta y \geq (\Delta y)_c$, the single chain bifurcates and forms a **zigzag** structure between nearest neighbours.

Molecular Dynamics Results

- Results show hysteresis is present in zero-temperature simulations. This indicates the transition occurs on a long time scale.



Numerical Techniques

- The vortex-vortex interaction is a modified Bessel function. If we rescale $a_0 \rightarrow 1$, we can approximate the potential using a cut-off length, r_c [3].

$$\begin{aligned} V\left(r\right) &= K_0\left(\tfrac{r}{\lambda}\right) \\ &\simeq \log\left(\tfrac{r_c}{r}\right) + \left(\tfrac{r}{r_c}\right)^2 - \tfrac{1}{4}\left(\tfrac{r}{r_c}\right)^4 - \tfrac{3}{4} \end{aligned}$$

- We simulate the system using Runge-Kutta algorithms to solve the overdamped **Langevin equation** of motion [4].

$$\eta\frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t} = \sum_j \mathbf{F}_{\text{vortices}}\left(|\mathbf{r}_i - \mathbf{r}_j|\right) + \mathbf{F}_{\text{drive}} + \chi\left(t\right)$$

Simulating Temperature

- We implement an **Andersen thermostat** [5] to act as a heat bath for the vortices. This will maintain a thermal distribution using a Markov process.

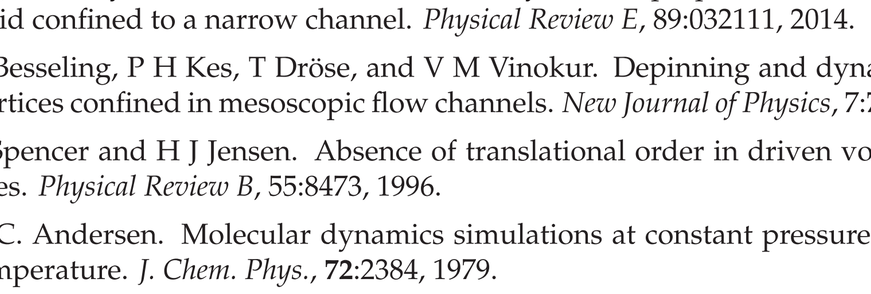
- The stochastic term in the Langevin equation gives a Gaussian distribution of thermal energies with two random uniformly generated numbers, $(u_1, u_2) \in (0, 1]$.

$$\chi(t) = \sqrt{\frac{2\delta t k_B}{\eta p}}\Theta(p - q_i)\sqrt{-2\log\left(u_1\right)}\left(\hat{\mathbf{x}}\cos\left(2\pi u_2\right) + \hat{\mathbf{y}}\sin\left(2\pi u_2\right)\right) \qquad (1)$$

Energetic Sampling

- We can find the **ground state** of the highly commensurate undriven system, to numerical precision, by sampling over different possible values of A_y and Δy. At widths greater than $(\Delta y)_c$ the energetic minimum is at a non-zero value of A_y indicating the transition has occurred.

- These results indicate the **single chain becomes unstable** beyond the transition point. If thermal fluctuations are sufficiently small, the transition will occur on a **long time scale**. In the complete absence of fluctuations, the single chain will never bifurcate.



Acknowledgements & References

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References

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