Operational models for ice crystal formation in highly concentrated systems

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Crystallisation

- ✓ important in food processing **freezing & freeze-drying**
- ✓ determining in creating microstructure (food properties and texture)
- ✓ challenging in **high concentrated systems** available water?

Why high concentrated systems?

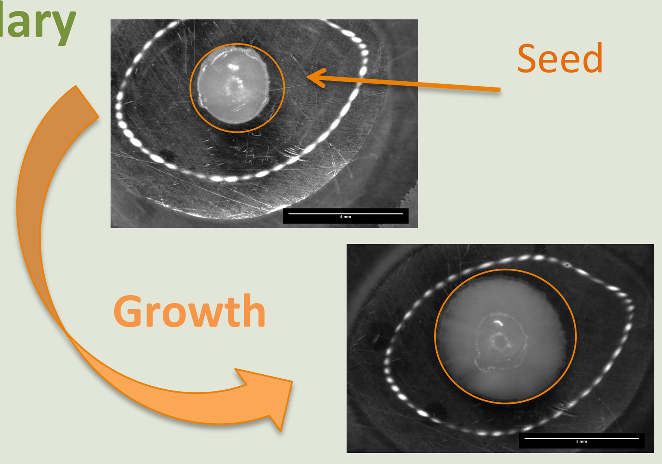
Less water → Less energy consumption

Experimental work

60% sucrose solution

Two crystallisation mechanisms

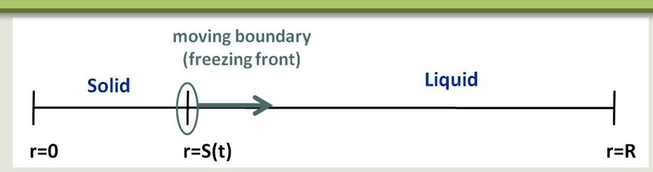
- **Secondary**



- **Primary homogenous**
  Differential Scanning Calorimetry (DSC): crystal fraction α

Secondary (seeding) model [1]

- Governing equations



$$\frac{\partial c_i}{\partial t} = \nabla\left(D_i \nabla c_i\right), \quad i = l,s \qquad \rho_i C_{p_i}\frac{\partial T_i}{\partial t} = \nabla\left(k_i \nabla T_i\right), \quad i = l,s$$

- Moving front boundary conditions

$$\left[c_{|S(t)^+} - c_{|S(t)^-}\right]\frac{\partial S}{\partial t} = D_l \left.\frac{\partial c}{\partial x}\right|_{S(t)^+} - D_s \left.\frac{\partial c}{\partial x}\right|_{S(t)^-}$$

$$\Delta H \rho_s \frac{\partial S}{\partial t} = -k_l \left.\frac{\partial T}{\partial x}\right|_{S(t)^+} + k_s \left.\frac{\partial T}{\partial x}\right|_{S(t)^-} ; T_i\left(S(t),t\right) = T_f^* - \Delta T$$

Freezing depression [3]

- External boundary conditions

$$T_s(0,t) = T_c < T_i\left(S(t),t\right); \ \frac{\partial T_l}{\partial r}(R,t) = 0 \qquad c_s(0,t) = c_{seed}; \ \frac{\partial c_l}{\partial r}(R,t) = 0$$

Homogeneous crystallisation model [2]

- Governing equations

$$\rho_m C_{p_m}\frac{\partial T_m}{\partial t} = \nabla\left(k_m \nabla T_m\right) + \rho_s \Delta H \frac{\partial \alpha}{\partial t}$$

- External boundary conditions

$$T_s(0,t) = T_c < T_i\left(S(t),t\right); \ \frac{\partial T_l}{\partial r}(L,t) = 0$$

- Continuous material properties

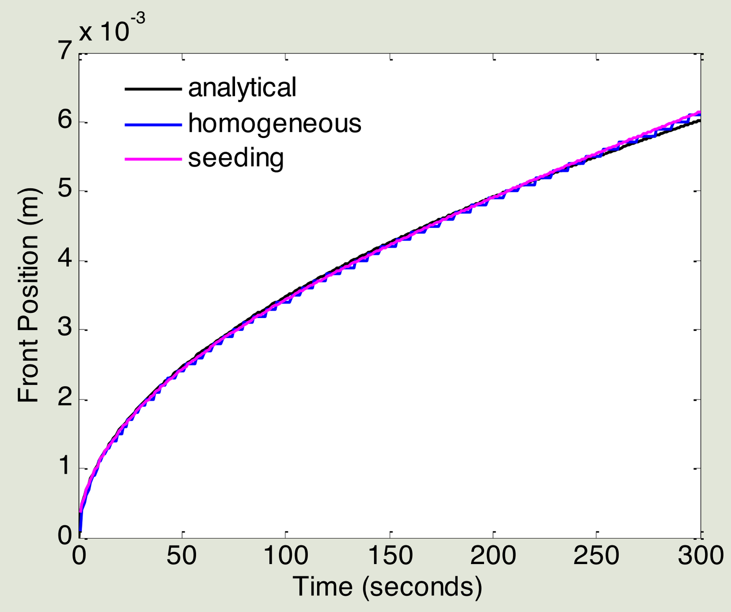
$$k_m = \varepsilon k_{air} + (1-\varepsilon)\left(\alpha k_s + [1-\alpha]k_l\right)$$
$$C_{p_m} = \varepsilon C_{p_{air}} + (1-\varepsilon)\left(\alpha C_{p_s} + [1-\alpha]C_{p_l}\right)$$
$$\rho_m = \varepsilon \rho_{air} + (1-\varepsilon)\left(\alpha \rho_s + [1-\alpha]\rho_l\right)$$

Air fraction (ε); Crystal fraction (α)

Numerical methodology & Results

Model validation

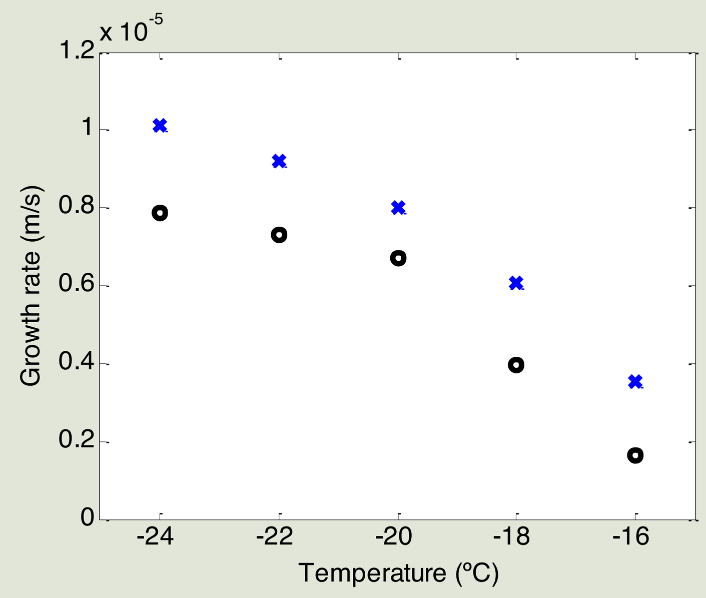
- Pure water, thermal problem
- Seeding: FEM+ALE (adaptive mesh)
- Homogeneous: FEM
- 101 nodes, tol=10^{-6}



Comparison of the freezing front position evolution in time between the analytical and the proposed models solutions

Seeding results

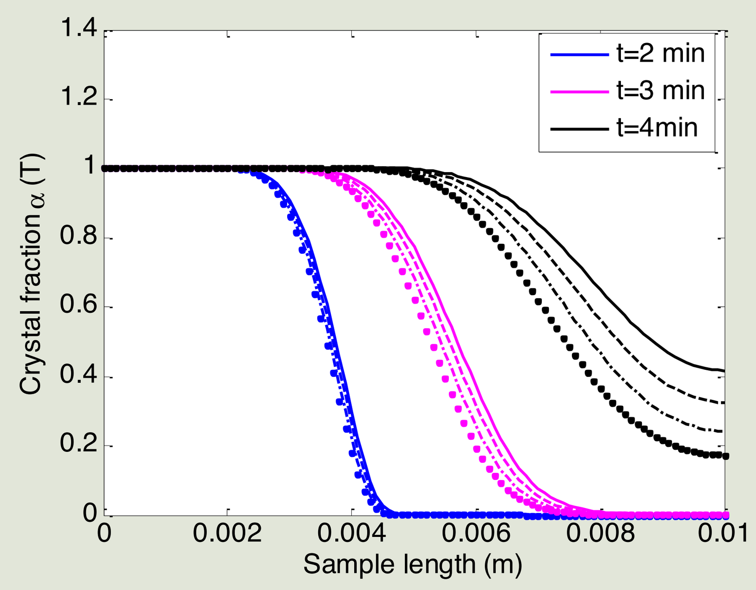
- 60% sucrose solution
- FEM+ALE (adaptive mesh) in COMSOL
- 101 nodes, tol=10^{-6}, R=1 cm



Comparison of experimental (o) and simulated (x) growth crystal rates for different cooling conditions

Homogeneous crystallisation results

- 60% sucrose solution
- FEM in COMSOL
- 101 nodes, tol=10^{-6}, L= 1cm
- Air fraction ε = [0, 0.1, 0.2, 0.3]



Comparison of the crystal fraction along the sample between the non-aerated 60 % w/w sucrose solution (solid), and the aerated solution with air fractions (dashed), (dashed-dot) and (dot)

Conclusions

- ➢ **Overall good agreement between models and experiments.**
- ➢ **Seeding model overestimates growth rates.**
- ➢ **Aeration affects heat transfer delaying ice crystal formation.**
- ➢ **Satisfactory first approach to modelling of high concentrated systems.**

Acknowledgements

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REFERENCES
[1] *Free and moving boundary problems* (1984), Clarendon Press, Oxford, UK.
[2] *International Journal of Refrigeration* (2010) **33**, 1559-1568.
[3] *Phase transitions in foods* (1995). Academic Press, London.