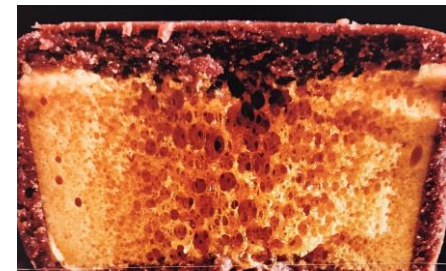


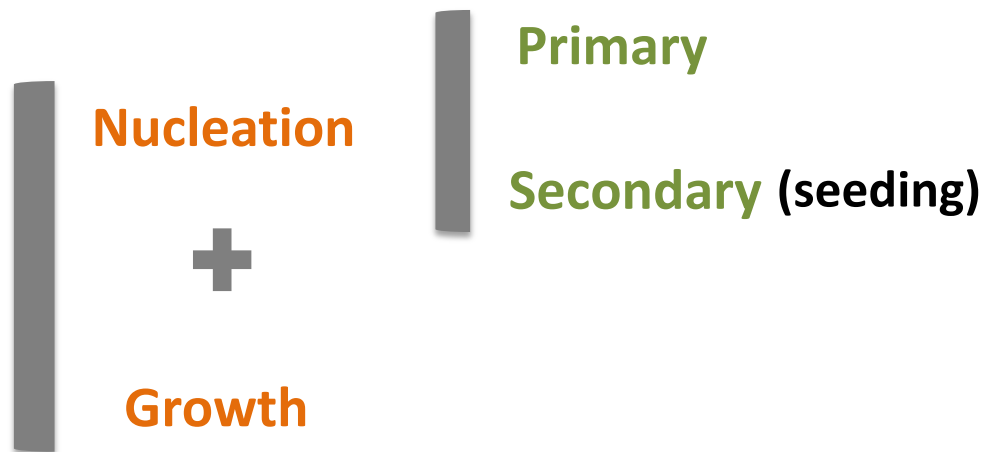
Operational models for ice crystal formation in highly concentrated systems

E. López-Quiroga, P.J. Fryer, O. Gouseti, R. Wang and S. Bakalis
School of Chemical Engineering, University of Birmingham, UK.

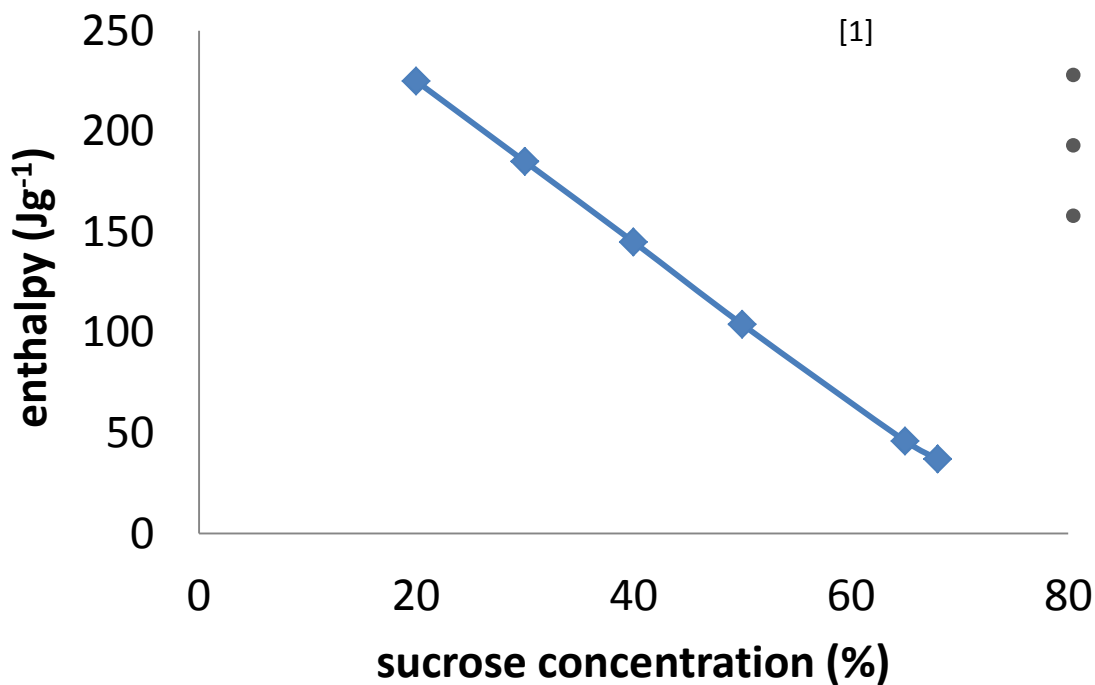
- Important in food processing → **freeze-drying & freezing**
- Determinant in creating microstructure → **properties & texture**



Crystallisation



Highly concentrated systems



- Experimentally challenging
- Available water?
- Difficult crystallisation

LESS WATER



LESS ENERGY

Modelling: understanding and control (Quality, process-ability)

- Liquid into solid → Phase change

Moving boundary problem

Primary^[2]

Kinetics:

DSC experiments

Heat transfer: Fourier
+ phase change heat

crystal fraction, T

Fixed domain

Secondary^[3]

Mass transfer:

Fick

Heat transfer:
Fourier

Stefan problem
Front-tracking

T , C , freezing front

Growth rate



Mean crystal size

[2] Chégnimonhan *et al.* (2010). *International Journal of Refrigeration* 33, 1559-1568.

[3] Crank (1984). *Free and moving boundary problems*, Clarendon Press, Oxford, UK.

Primary crystallisation: model

- Governing equations

$$r_m C_{p_m} \frac{\partial T_m}{\partial t} = \nabla \cdot (k_m \nabla T_m) + r_s DH \frac{\partial a}{\partial t} \rightarrow \text{Crystal fraction}$$

- External boundary conditions

$$T_s(0, t) = T_c < T_{freezing} ; \frac{\partial T_l}{\partial r}(L, t) = 0$$

- Continuous material properties

Air fraction \leftarrow

$$k_m = e k_{air} + (1 - e) (a k_s + [1 - a] k_l)$$

$$C_{p_m} = e C_{p_{air}} + (1 - e) (a C_{p_s} + [1 - a] C_{p_l})$$

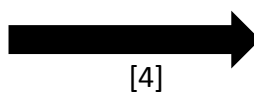
$$r_m = e r_{air} + (1 - e) (a r_s + [1 - a] r_l)$$

Primary crystallisation: model

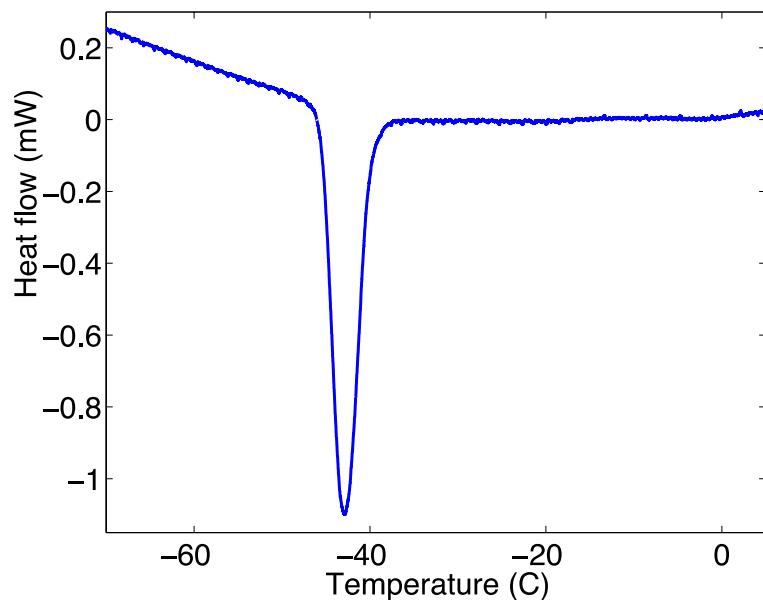
- Crystallisation kinetics

$$\alpha(T) = \frac{1}{\Delta H \dot{q}} \int \phi dT$$

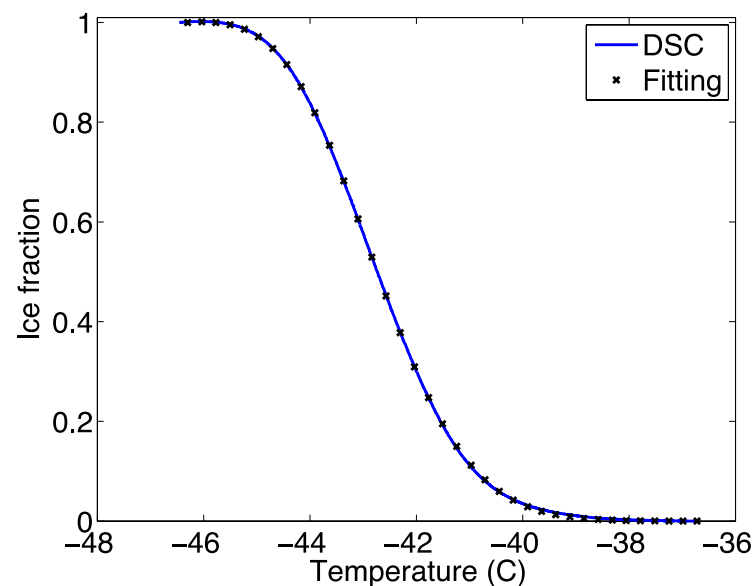
DSC experiments



α - Ice crystal fraction



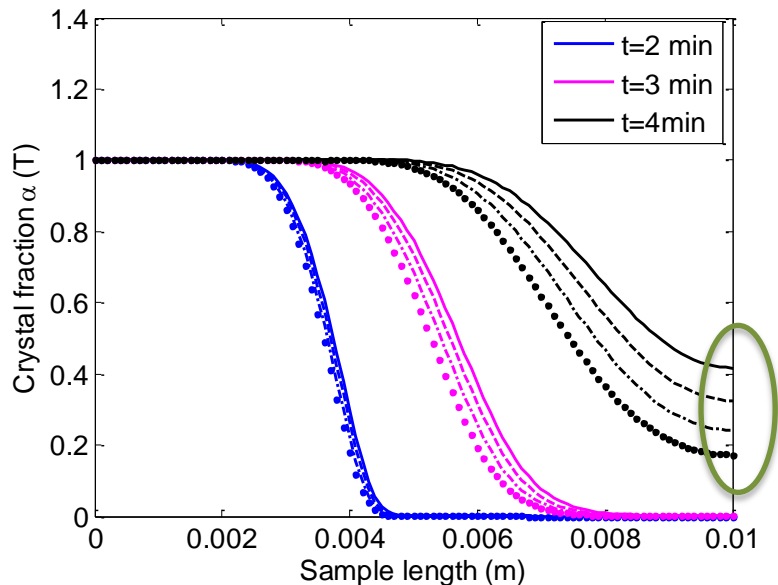
Non-isothermal DSC thermogram for the 60% (w/w) sucrose solution



Crystallisation rate for the 60% (w/w) sucrose solution
RMSE = 0.001945

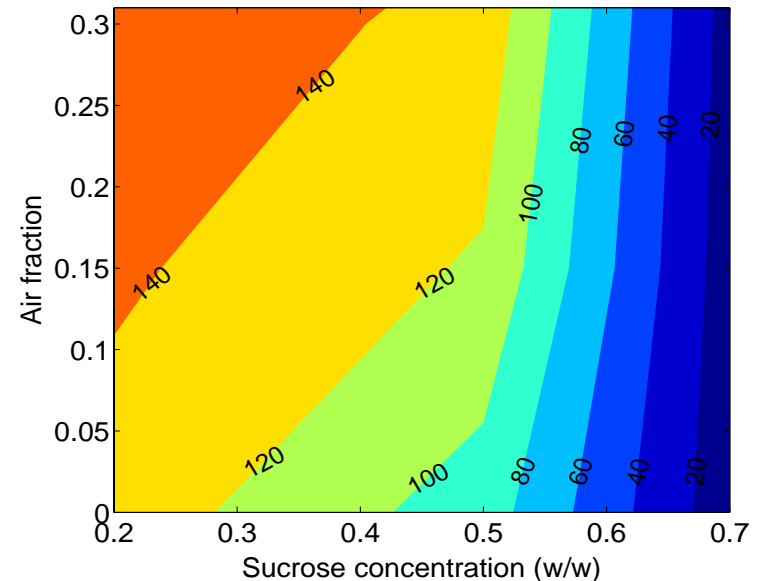
Primary crystallisation: results

- 20-70% sucrose solution
- FEM in COMSOL
- 101 nodes, $\text{tol}=10^{-6}$, $L=1\text{cm}$
- Air fraction $\varepsilon = [0, 0.1, 0.2, 0.3]$



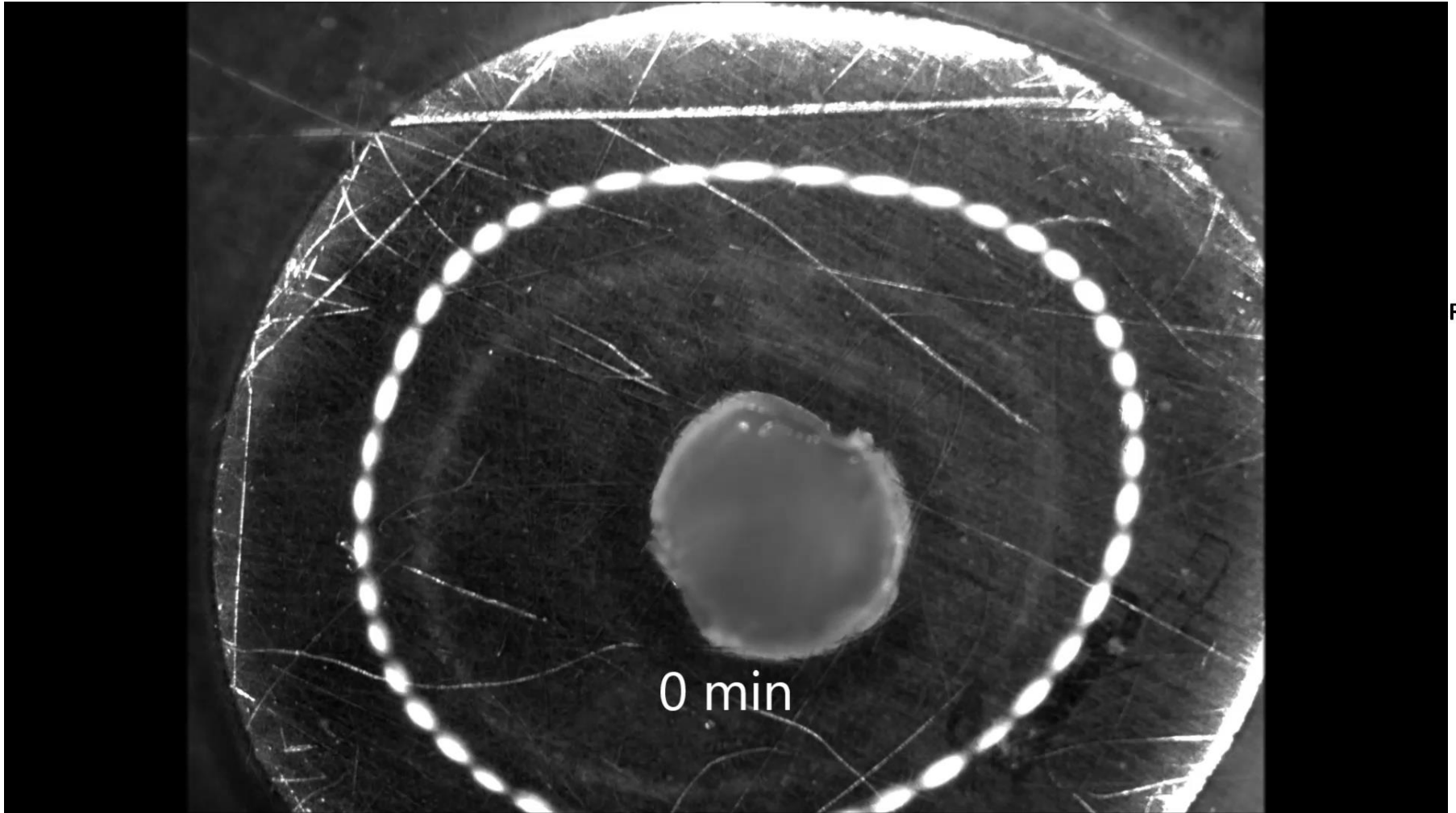
Crystal fraction the for the % 60 non-aerated sample (solid), and the aerated system with air fractions $\varepsilon=0.1$ (dashed), $\varepsilon=0.2$ (dashed-dot) and $\varepsilon=0.3$ (dot)

Air bubble size



Effect of product formulation and aeration on the mean ice crystal size (μm).

Secondary crystallisation: experiments



Secondary crystallisation: model


- Governing equations

$$\frac{\partial c_i}{\partial t} = \nabla \left(D_i \nabla c_i \right), \quad i = l, s \qquad r_i C_{p_i} \frac{\partial T_i}{\partial t} = \nabla \left(k_i \nabla T_i \right), \quad i = l, s$$

- Moving front boundary conditions

$$\left[c \Big|_{S(t)^+} - c \Big|_{S(t)^-} \right] \frac{\partial S}{\partial t} = D_l \frac{\partial c}{\partial x} \Big|_{S(t)^+} - D_s \frac{\partial c}{\partial x} \Big|_{S(t)^-}$$

$$DH r_s \frac{\partial S}{\partial t} = -k_l \frac{\partial T}{\partial x} \Big|_{S(t)^+} + k_s \frac{\partial T}{\partial x} \Big|_{S(t)^-} ; T_i(S(t), t) = T_f^* - DT$$

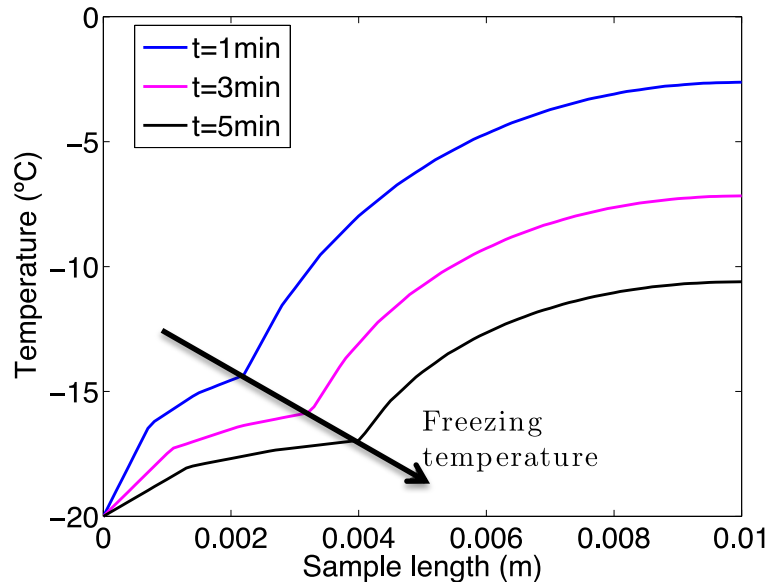
Freezing depression [5] 

- External boundary conditions

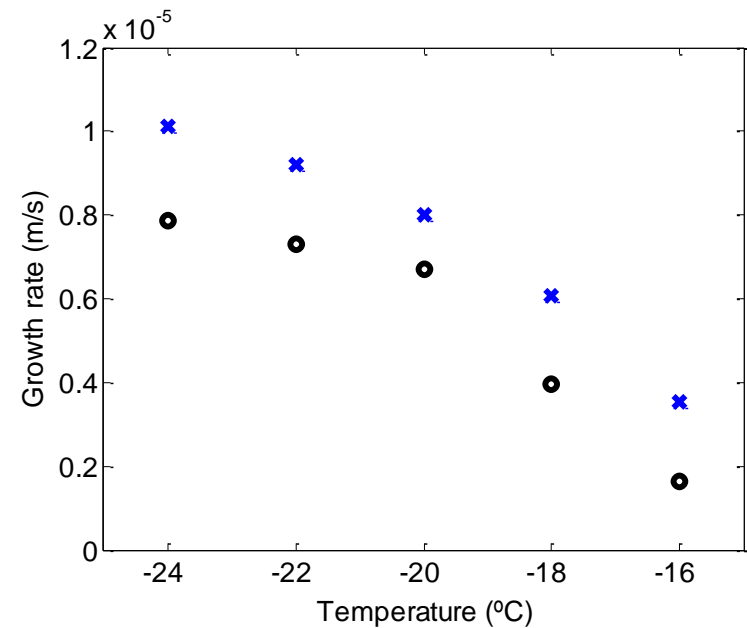
$$T_s(0, t) = T_c < T_i(S(t), t); \quad \frac{\partial T_l}{\partial r}(R, t) = 0 \qquad c_s(0, t) = c_{seed}; \quad \frac{\partial c_l}{\partial r}(R, t) = 0$$

Secondary crystallisation: results

- 60% sucrose solution
- FEM+ALE (adaptive mesh) in COMSOL
- 101 nodes, $\text{tol}=10^{-6}$, $R=1$ cm



Temperature distribution along the sample during the seeding simulation with $T_c = -20$ °C .



Comparison of experimental (o) and simulated (x) growth crystal rates for different cooling conditions

- **First approach to modelling of high concentrated systems**
- **Overall good agreement between models and experiments:**
 - seeding model overestimates growth rates.
 - aeration affects heat transfer delaying ice crystal formation.

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