Analysis and simulation of various Stewart Platform configurations for lower limb rehabilitation

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Abstract

In this paper the structure of five different types of Stewart platform were compared to one another with regard to stability under different load conditions. These include a platform with semi-regular hexagons (PSRH), a triangular simplified symmetric manipulator (TSSM), a minimal simplified symmetric manipulator (MSSM), a platform with regular hexagons (PRH) and a platform with similar symmetric hexagons (SSH). Matlab and SolidWorks software were used to determine the exact position of the upper platforms, the length of actuators, and the workspace of the platforms through inverse kinematic analysis following a specific motion profile at a particular time. The desired trajectory according to the workspace of the platforms were also determined. The results show that the platform with semi-regular hexagons has the most stable structure of all. The results are then considered for a robotic lower limb rehabilitation system.

1 Introduction

The aim of physiotherapy by robots is to move paralyzed and disabled organs because of diseases such as cerebral apoplexy (Speich *et al.*, 2004).Using the mobile platform under the patient's foot in order to create movement is a mechanism which is used in the rehabilitation of the ankle. In this mechanism, by programming the movement of the mobile platform, the entire foot or some parts of the foot are impelled to move.

With respect to different arrangements of the joints on platforms and the shape of the platforms, there are different structures with varying levels of stability. In this study five Gough –Stewart platforms of different structures were investigated; one of them is the PSRH structure (a platform with semi-regular hexagons); all of its base joints and all its mobile joints are coplanar and the distance between the actuators is so much greater than in other structures that the probability of contact between the actuators is reduced. Another structure is that of the TSSM (Triangular simplified symmetric manipulator); the mobile platform in this structure is triangular and the joints of the upper platform are close to each other. The third structure is called the SSH (Similar symmetric hexagons); here the shape of the mobile and fixed platforms is symmetric and similar. The structure of similar symmetric hexagons is much like that with semi-regular hexagons, except that with similar symmetric hexagons, the platforms are related to each other in shape ratios of the same size. The fourth structure, called the PRH (Platform with regular hexagons) has regular and similar hexagonal mobile and fixed platforms. The last configuration, called the MSSM (Minimal simplified symmetric manipulator), combines the TSSM with a 3-3 platform, and its mobile and fixed platforms have a similar triangular shape (St-Onge *et al.* 2000 , Ahmadi *et al.2013*).

It is very complex to identify all the foot motions because these are on three planes and axes. Three kinds of movement are defined for the foot, referred to as plantar\dorsiflexion, eversion\ inversion, and adduction\abduction. According to an analysis of the foot anatomy which was performed in the

rehabilitation centre, the desired extent of translation Motions along x, y and z directions is 30 cm, 5-15 cm and 20cm, respectively (Saglia *et al.,*2009).
To choose the most suitably structured Stewart platform for rehabilitation applications, one must consider the anatomy of the ankle and the stability of the five different Stewart platform structures, as analyzed by Solidworks software. Next, the kinematic of the Stewart platform for all configurations was considered and on this basis the maximum workspace of the selected robot was examined.

2 Methodology

2.1 Finite Element Analysis (FEA)

As it can be seen in Figure 1, using Solidworks software, presents the Finite Element analysis (FEA) of the Stewart platform with different structures, in order to find the most stable structure to use for purposes of in rehabilitation. The foot gripper which fixes the foot on top of the platform has been used to perform dorsiflexion and plantarflexion of ankle. In this analysis; assumptions are made about the nature of the problem and the computational limits. All the structures were designed and analyzed statically in the home position, when three different forces – 1500 N, 1300 N and 1100N – were exerted vertically on the gripper. The uniform standard mesh size, small in scale and of high quality, is used for finite element analysis. After finite element analyses had been made of the different structures, it was found that by exerting different forces, the PSRH structure had the lowest stress and displacement, with a higher factor of safety and greater stiffness than the other structures.

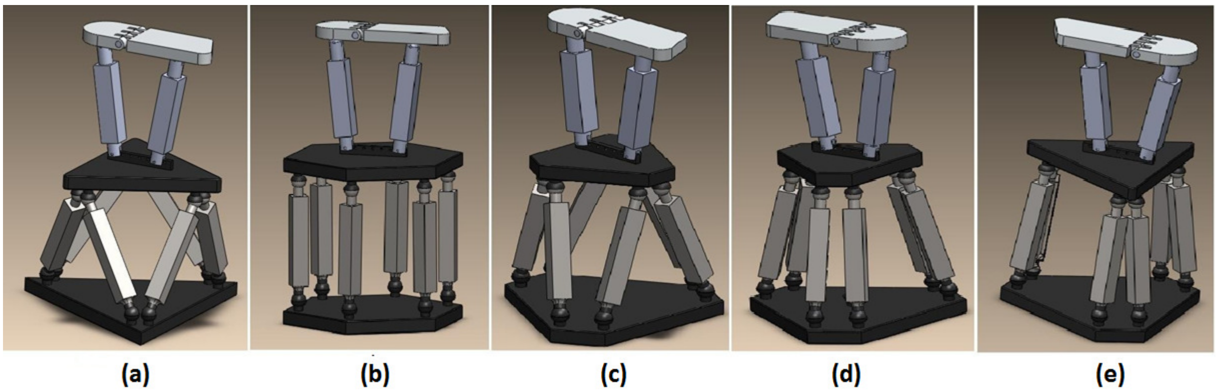

Figure 1-Five different configurations of parallel robot.(a) MSSM ,(b) PRH ,(c) PSRH,(d) SSH,(e) MSSM

2.2 Kinematic

It is worth mentioning that singularity occurs on PRH because of identical geometry of lower platform and upper platrform,equal length of actuators and parallel configuration of actuators and PRH can not be considered as a rehabilitation robot. In all the configurations, the lower platform and upper platform are called respectively the “Base” and the “Top”. The linear actuators, which will be called “legs”, are connected to the vertices of the Base and Top with spherical joint (Liu 1993). With respect to **Gruebler's Equation**, this gives the whole system six degrees of freedom (Tsai 1996).The coordinate system (X, Y, Z) was placed on the centre of the base with the *Z* axis pointing vertically upwards, called the base frame. The end effecter coordinate system (x, y, z) was attached to the centre of the end effecter platform with the z axis perpendicular to the end effector, pointing upwards; this coordinate was called the top frame (Gonzalez *et al*,.2011).

L1, L2,..., L6 represent the lengths of the actuators, $[px \quad py \quad pz]^T$ represent the location of the top frame with respect to the base frame and (α , β , γ) represent the rotation angles of the top frame , first around the X axis to α degrees, then around the Y axis to β degrees and at last around the Z axis to γ degrees. Therefore the position and orientation of the upper platform is determined by:

$$Xp = [px, py\ pz\ , \alpha, \beta, \gamma]^T \qquad (1)$$

The coordinates of these Points with respect to the Base frame are known and fixed, as can be seen in Figure 2. For the five different configurations, the coordinates of vertices of the base and top, with respect to the Base frame, were calculated.

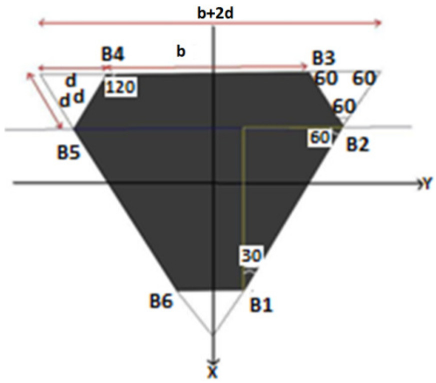

Figure 2 -The hexagonal platform

As Figure 2 shows, in PSRH structure**,** the hexagon is inscribed by an equilateral triangle whose sides are (b+2d) long. The coordinate system was placed in the centre of equilateral triangle whose *x* axis divided the side B1B6 into two equal parts (St-Onge 2000). With respect to this coordinate system, the coordinates of the hexagon vertices were calculated. The side of b,d,B and D were respectively 20cm,66.67cm,30cm and 10cm respectively .For example the coordinate of 2 vertices in 2 dimension is calculated and other vertices can be calculated with respect to geometry of the Top and Base platforms .

$$\text{Height of the equilateral triangle} = \frac{\sqrt{3}}{2}(B + 2D) \qquad (2)$$

$$XB1 = \frac{\sqrt{3}}{6}(2B + D) \qquad (3)$$

$$XT2 = \frac{\sqrt{3}}{6} \qquad (4)$$

$$YT2 = \frac{(b+d)}{2} \qquad (5)$$

In the platform with a structure of regular hexagons, the coordinates of the vertices of the top and base are the same, but the size of the sides of the base differs from the size of the sides of the top. The base side and Top side were 20cm and 30cm, respectively. For example the coordinate of a vertex calculated based on geometry of the platform;

$$\mathrm{XB1} = \frac{\sqrt{3}}{2}\mathrm{a} \qquad (6)$$

$$\mathrm{YB1} = \frac{\mathrm{a}}{2} \qquad (7)$$

In the TSSM structure, the coordinates of base vertices are similar to those of the semi regular hexagons. The side of Top (a), bigger side of the Base and smaller side of the Base were 20cm, 30cm and 10cm, respectively. For instance the coordinate of one of the vertices calculated here:

$$XT1 = \frac{\sqrt{3}}{6} a \qquad (8)$$

In the MSSM structure, because both platforms are triangular in shape, the coordinate of their vertices are the same as the coordinates of the vertices of the upper platform in the TSSM structure with sides of different sizes.The side of the Top was 20cm and side of the base was 30cm.

In the similar symmetric hexagons configuration, both platforms have a similar shape but the sides are of different sizes. So, the coordinates of the vertices of both platforms, with the different sizes of their sides, would be same as the coordinates of the vertices of the lower platform of the semi-regular hexagons. The side of b,d,B and D were respectively 20cm,66.67cm,30cm and 10cm respectively. After calculating the geometry of each structure, the homogeneous transformation matrix from the TOP (upper platform) to the BASE (lower platform) frames was used for all of the configurations to find the position of the upper platform. It is described by the following transformation matrix:

$$T_{BASE}^{TOP} = \begin{bmatrix} \cos\beta\cos\gamma + \sin\alpha\sin\beta\sin\gamma & -\cos\beta\sin\gamma + \sin\alpha\sin\beta\cos\gamma & \cos\alpha\ \sin\beta & Px \\ \cos\alpha\sin\gamma & \cos\alpha\cos\gamma & -\sin\alpha & PY \\ \sin\beta\cos\gamma + \sin\alpha\cos\beta\sin\gamma & \sin\beta\sin\gamma + \sin\alpha\cos\beta\cos\gamma & \cos\alpha\cos\beta & PZ \end{bmatrix} \qquad (9)$$

Where α,β and γ are rotational motion around X,Y and Z axis, respectively and Px,Py and Pz are translations in X, Y and Z direction. With respect to this homogeneous transformation and the trajectory of *Xp*, the coordinates of the vertices of the top and base for all of the configurations were calculated by the following equation: (Zhang et al., 2002)

$$\begin{bmatrix} XTi \\ YTi \\ ZTi \\ 1 \end{bmatrix} = T_{BASE}^{TOP}(px\, , py\ pz\ \ , \alpha, \beta, \gamma) \begin{pmatrix} X_{Ti} \\ Y_{Ti} \\ Z_{Ti} \\ 1 \end{pmatrix} \qquad (10)$$

The vertices coordinates of the upper platform and the length of the actuators for all of the configurations were calculated by following equations. But because of the long and bulky equations and repetitive method of calculation, only the equations for calculating the one of the vertices of the top and length of one of the actuators of PSRH structure are given (St-Onge 2000).

$$X_{T1} = -\frac{\sqrt{3}}{6}(2b + d) \times (\cos\beta\cos\gamma + \sin\alpha\sin\beta\sin\gamma) + \frac{d}{2}(-\cos\beta\sin\gamma + \sin\alpha\sin\beta\cos\gamma) + + Px \qquad (11)$$

$$Y_{T1} = -\frac{\sqrt{3}}{6}(2b + d) \times \cos\alpha\sin\gamma + \frac{d}{2}\cos\alpha\cos\gamma + \ Py \qquad (12)$$

$$Z_{T1} = -\frac{\sqrt{3}}{6}(2b + d) \times \sin\beta\cos\gamma + \sin\alpha\cos\beta\sin\gamma + \frac{d}{2}\sin\beta\sin\gamma + \sin\alpha\cos\beta\cos\gamma + \ Pz \qquad (13)$$

$$L1 = \sqrt{(x_{T3} - \frac{d}{2\sqrt{3}} - \frac{b}{\sqrt{3}})^2 + (y_{T3} - \frac{d}{2})^2 + \ z_{T3}{}^2} \qquad (14)$$

$$L1 = T_3 - B_1 \qquad (15)$$

Where, X_{T1} , Y_{T1} and Z_{T1} representing the coordinate of the point 1 in Top platform and L1 corresponds to length of actuator number 1 .

2.3 Path motion

To define an accurate path motion for rehabilitation of ankle ,gait analysis have been performed in West Midland Rehabilitation Centre (WMRC), UK. Twenty able-bodied participated in this experiment with age(year) of 24.34±4.83,weight(kg) of 73.41±5.2,height(cm) of 172.74±4.2. As shown in Figure 3, the range of motion and trajectory of movement for lower limb joints have been analysed by Vicon Nexus software.

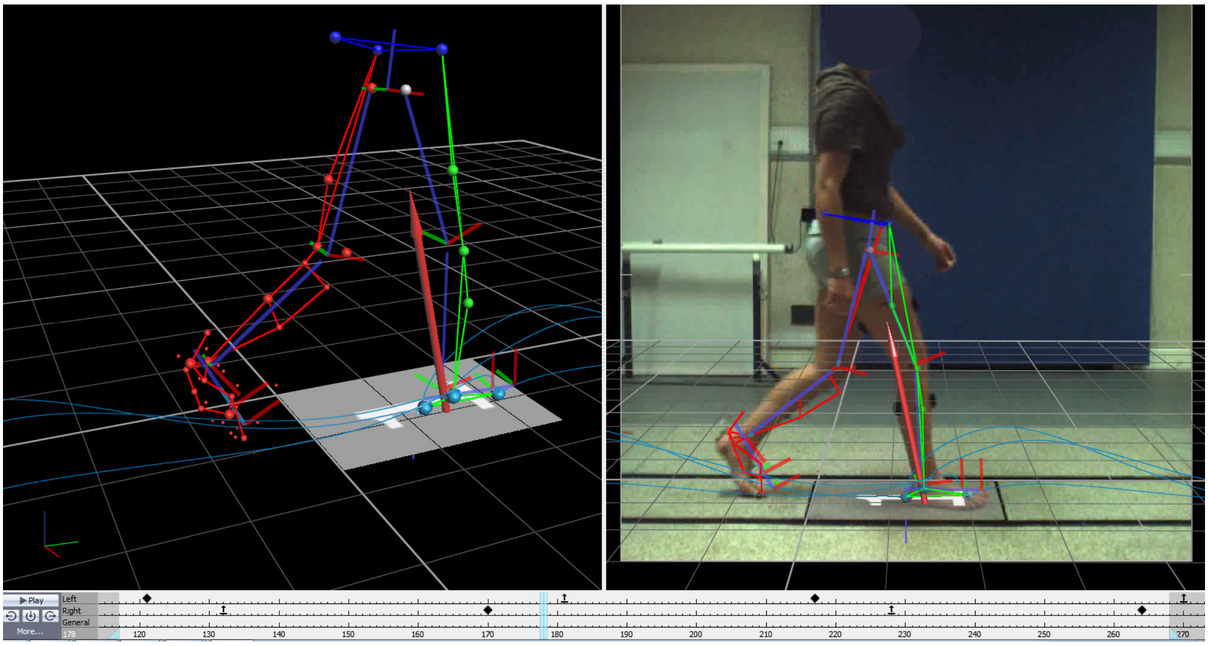

Figure 3 - Gait analysis

Once all the lengths of the coordinates and actuators were calculated, the whole algorithm with some modifications was imported to Matlab software for simulating different motions. In this algorithm, by defining the specific motion profile, orientation and time domain, the exact position of the top (including the coordinates of the centre and vertices of the top) was found at different times.With respect to obtained data from gait analysis, the following trajectory has been defined for the movement of the Top platform, as an rehabilitation exercise:

$$P(i)_{(x,y,z,\alpha,\beta,\gamma)} = (\frac{i}{n} \times 68, \frac{i}{n} \times 35, \frac{i}{n} \times 15, 10, -5, -2) \qquad (16)$$

Where, n referred to the length of time domain, which is [0:25]; parameter (i) is used as a counter, which is in the domain of [2:25].

Workspace of the 6-Dof parallel robot programmed with Matlab software package based on the developed inverse kinematic formulation. The search engine points of workspace is based on Cartesian and polar algorithm. The length of the actuators for each specified position and orientation is calculated in equation 12 in order to identify the stroke size and possibility of each applied motion.

$$L_i = T_{BASE}^{TOP} \times T_i - B_i \qquad i \in \{1 \dots 6\} \qquad (17)$$

Where, L_i, T_i and B_i are the length of actuators, joint position on moving platform and joint position on base platform respectively.

3 Result

3.1 FEA results

Different forces, 1500 (N), 1300 (N) and 1100 (N), were exerted on the gripper and the maximum displacement and maximum stress for each structure were measured. In FEA analysis, the material which was used for upper platform, lower platforms and gripper was aluminium alloy 1345 and the material of the actuator shafts was carbon steel (Ashby 2005). The thickness of the upper platform and lower platform in all the structures was 3 cm. Moreover, in all the structures, spherical joints of the same size were applied to the upper and lower levels of the Stewart platform. All the structures had the same actuators, the stroke of which was fixed at 2 cm.

The joints play a crucial role in design and in order to achieve the desired DOF, this design used two types of spherical and prismatic joints. Aluminium alloy 1345 was chosen for the upper platform and lower platform (Ashby 2005). By exerting a force of 1500 (N), the PSRHs structure had the maximum displacement of 0.056 mm and maximum stress of 21 (Mpa). The primary stress analysis shows that the maximum stress for PSRH (2.1e10). By exerting a load of 1500 (N), the MSSMs structure tolerated the greatest stress of all the structures. PSRH had the highest factor of safety (1.31) and stiffness 26619 (N/mm) of all structures and these results lead to the choice the PSRH structure as the most stable one for manufacturing application.

3.2 Kinematics

The graphs of variation of the length of the actuators with respect to the time, for the five possible structures of the Stewart platform were calculated. With respect to the defined motion profile and initial length of the actuators 36cm and their utmost length 61cm, that the acceptable range of length of the actuators in the PSRH structure would be in the time domain of [19:21] and, in this time domain, the length of the actuators is between 36cm and 61cm. This means that, in other times in the time domain of 0:25, one or more actuators cannot reach the desired length.
With respect to the obtained data from gait analysis, the mean value for ankle`s trajectory of 20 participants have been analysed and trajectory of left ankle is shown in Figure 4 :

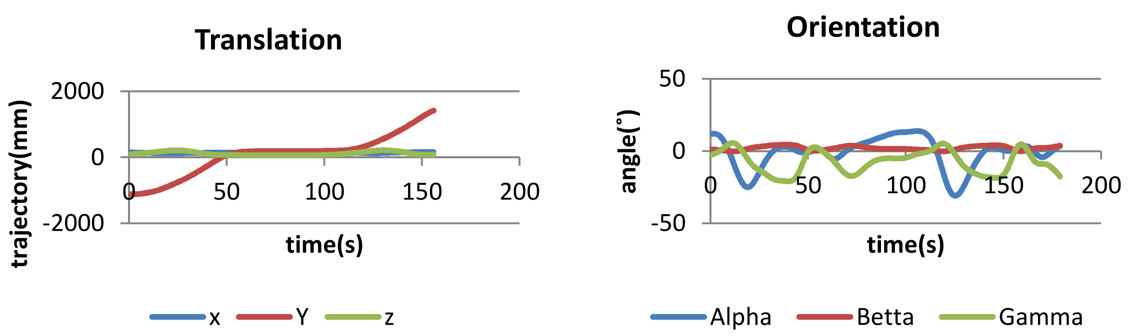

Figure 4 - Trajectory if the ankle joint in a gait cycle

As it can be seen in Figure 5, the path motion for all of the configurations have been simulated using Matlab software .

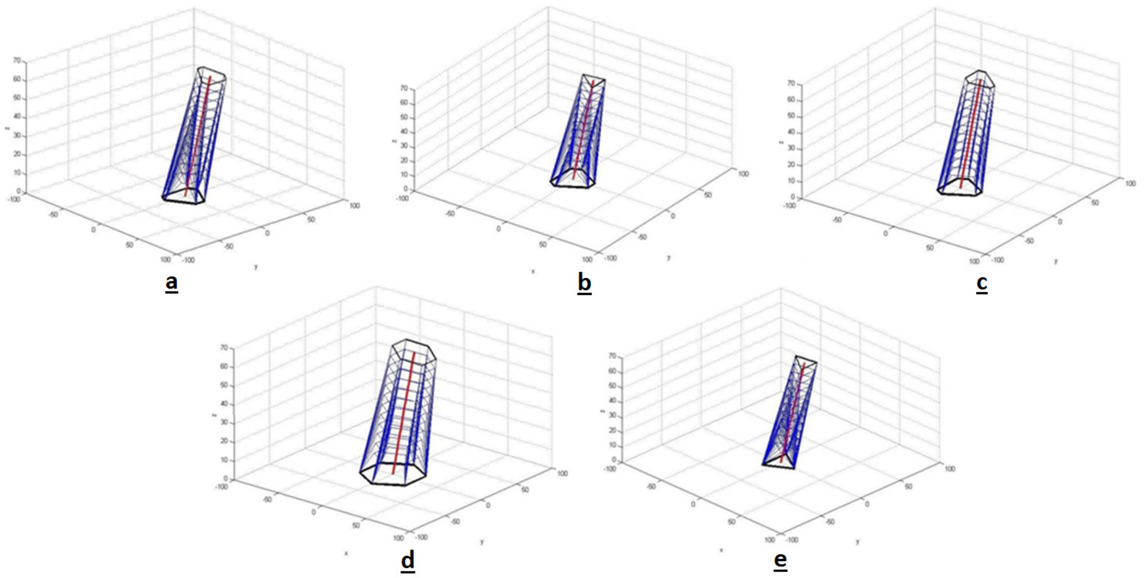

Figure 5 - The position of the Stewart platform with different structures in same motion profile.(a) PSRH; (b) TSSM; (c) SSH; (d) PRH; (e) MSSM

A suitable and simple motion profile with the desired orientation was selected and applied to the algorithm**.** Figure 6 shows that the workspace of the designed Stewart platform with the PSRH structure was measured and the shape of the derived workspace was similar to what was found by Stoughton (Stoughton *et al,*. 1993). Manipulators which have a short distance between joints such as the TSSM and MSSM have a minor workspace compared with manipulators which have a long distance between joints.

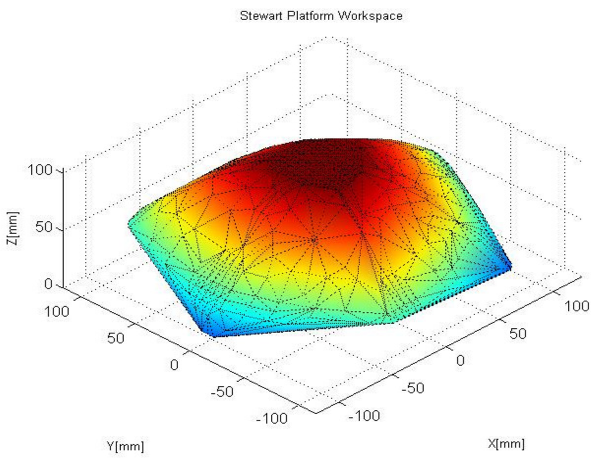

Figure 6 - Workspace Results of Hexapod

4 Conclusion

Different configurations of parallel robot have been modelled and with respect to FEA analysis, their stability compared with each other. Platform of semi regular hexagons showed more stability in compare with other structures and it was found that the stability of different structures completely depends on the position of the joints and actuators and the shape of the platforms. Then, with respect to the gait analysis, the range of movement of lower limb`s joints for 20 able-bodied was obtained.

With respect to kinematic analysis and gait analysis, a suitable path motion was defined as a rehabilitation exercise and based on kinematic analysis and defined path motion, the workspace of robot was calculated. With respect to FEA analysis, Kinematic analysis and range of motion of lower limb`s joints, it was found that PSRH structure is a suitable choice for rehabilitation purposes.

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