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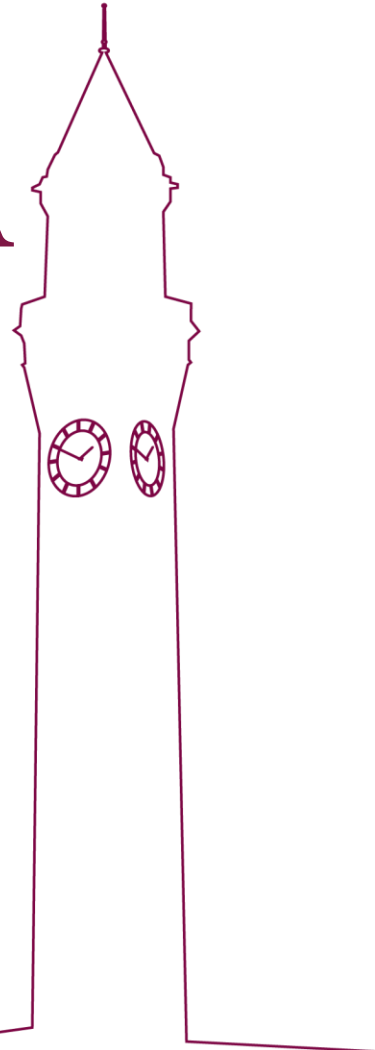
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Lagrangian Recurrences: A Novel Method for Mixing Description

Chiya Savari

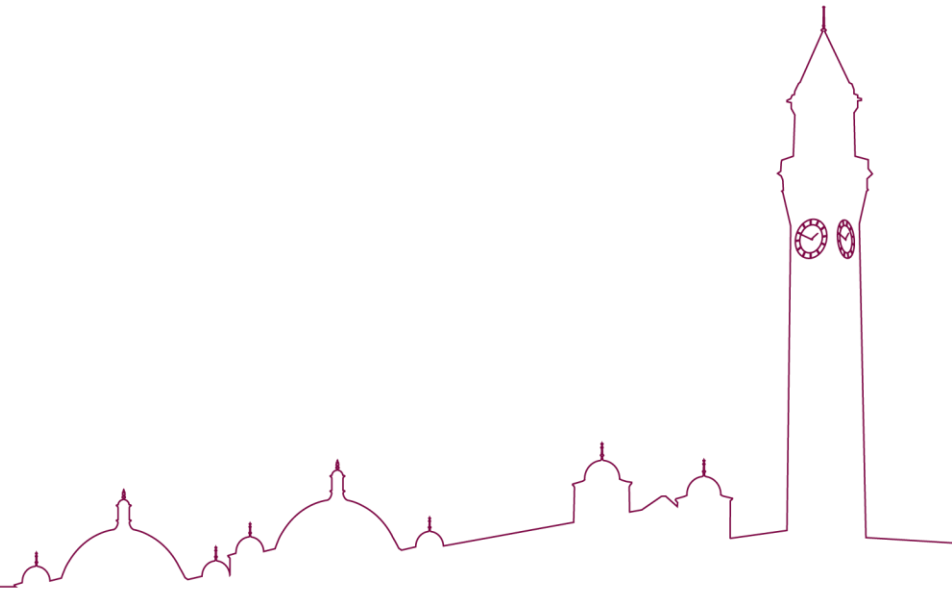
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Outline

- **Mixing**
- **Experimental**
- **Data Analysis**
- **Results**



Mixing

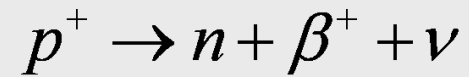
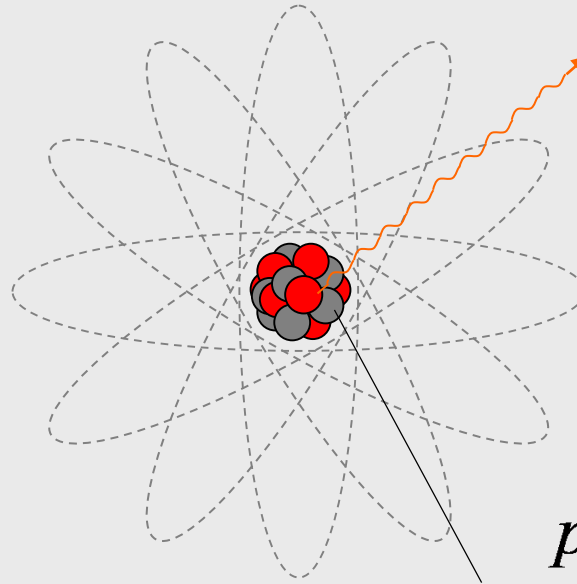
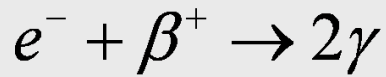
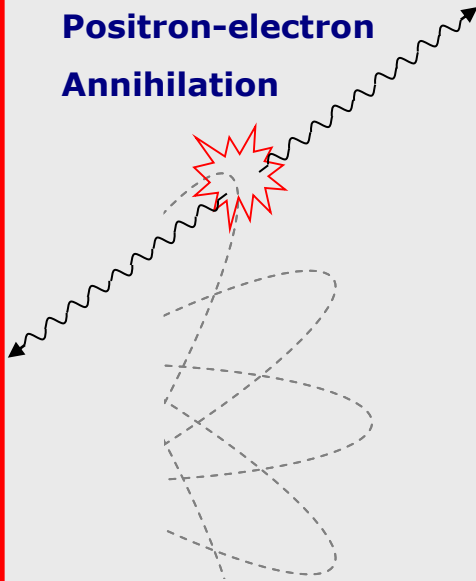
- Physical process which aims at reducing non-uniformities
- Fine chemicals, pharmaceuticals, personal/home care products, paper and pulp, polymers, food
- Traditional mixing performance indicators are generally based on Eulerian data
- Mixing is intrinsically a Lagrangian process
- Unique technique of positron emission particle tracking (PEPT)
- Compared with leading optical laser techniques (e.g. LDV, PIV)









Experimental Work

Atomic scale

Positron-electron
Annihilation



Unstable nucleus, proton-neutron
conversion occurring

-  Neutron n
-  Proton p^{+}
-  Electron e^{-}
-  Positron β^{+}
-  Neutrino ν
-  Photon γ

$$H = T$$

$$X = 0-40 \text{ wt}\%$$

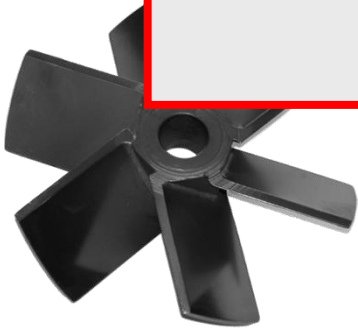
$$d_{\text{bead}} = 2.85-3.30 \text{ mm}$$

$$N = N_{js}$$

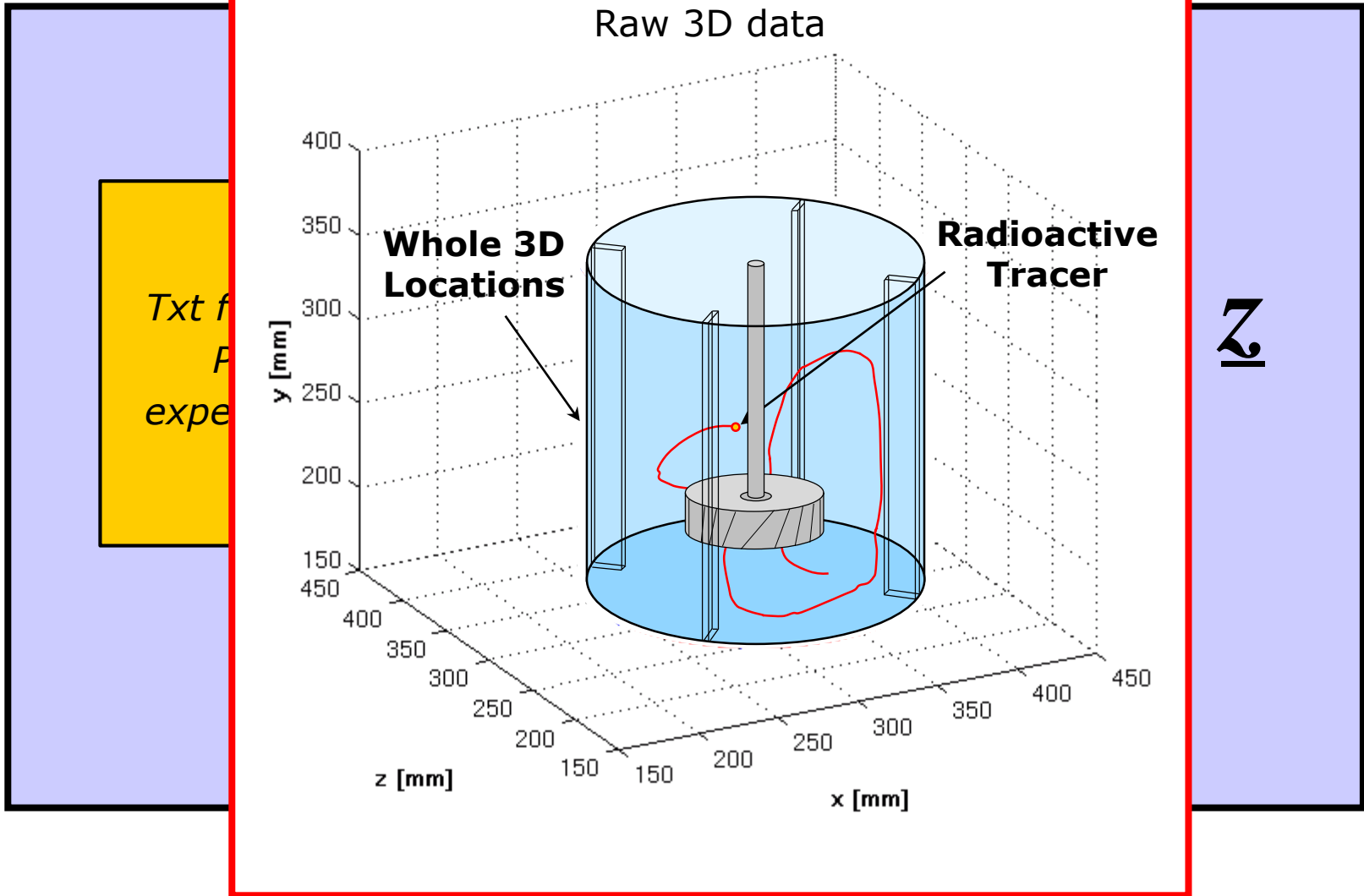
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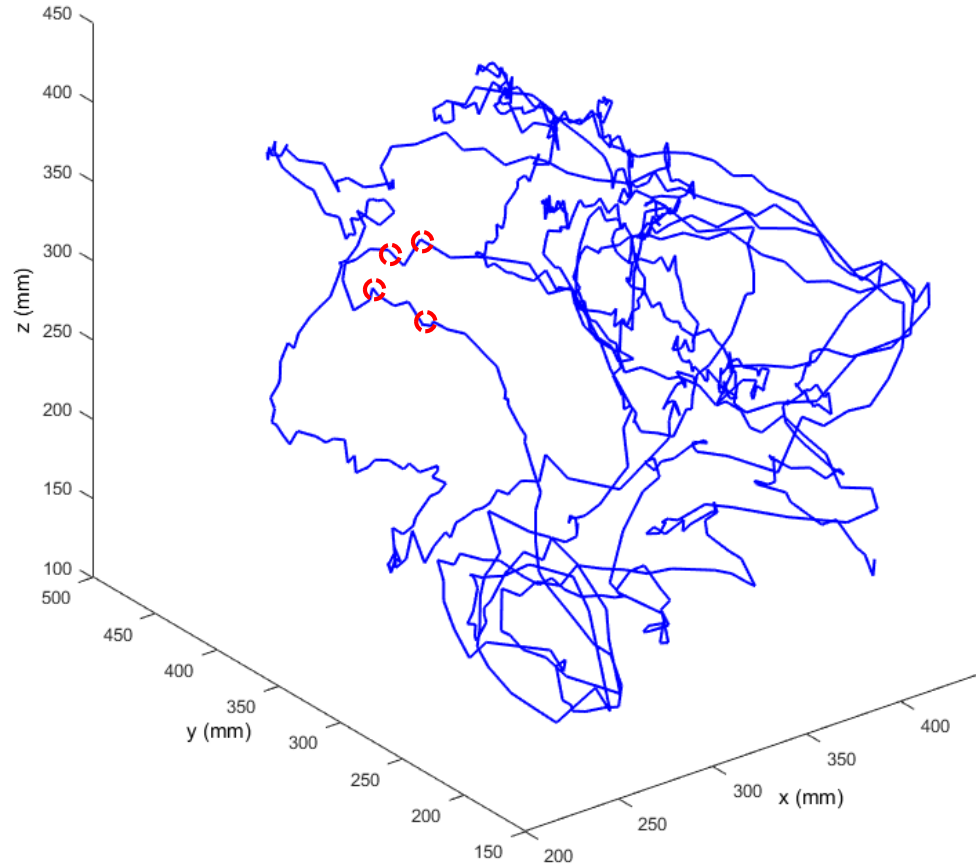


Data Analysis



Recurrence Matrix

- Distances between every points of trajectory can be calculated.
- For a trajectory of n data points, a matrix ($n \times n$) named distance matrix (DM) can be computed.
- **Euclidean distance** is the most common use of distance



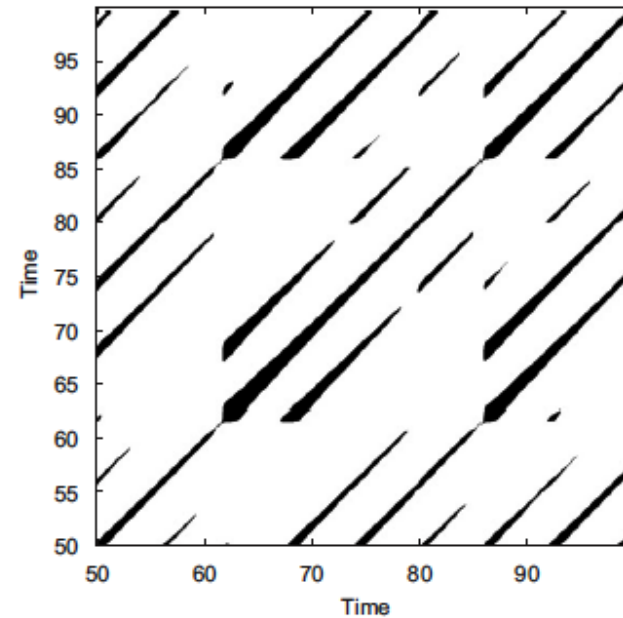
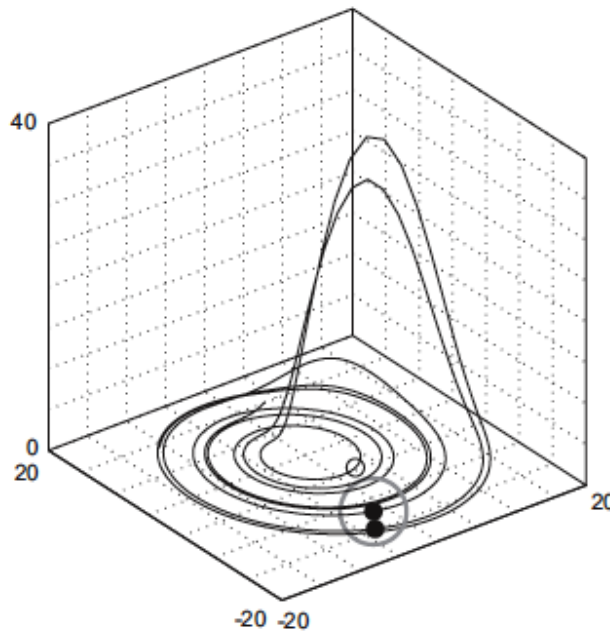
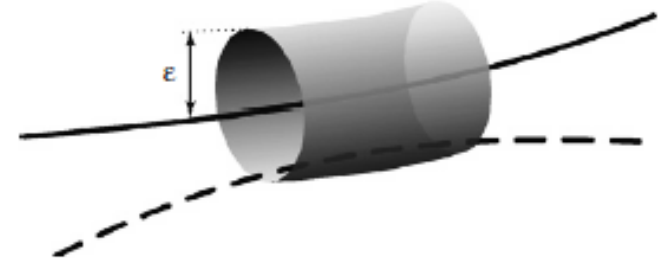
$$\text{dist}(a,b) = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2}$$



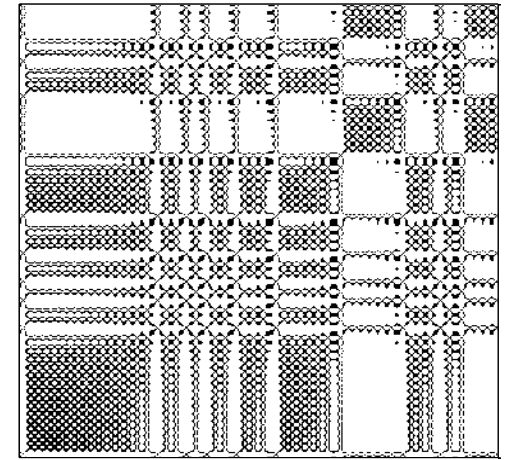
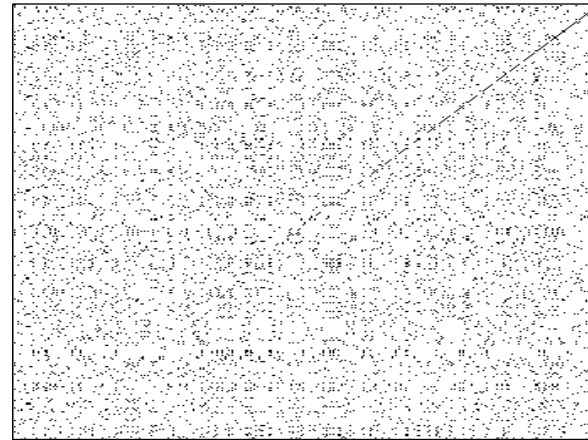
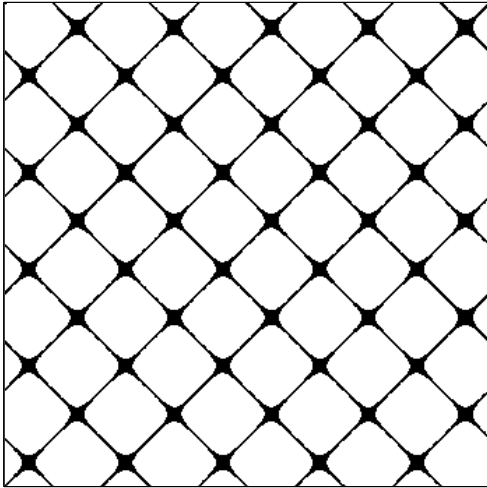
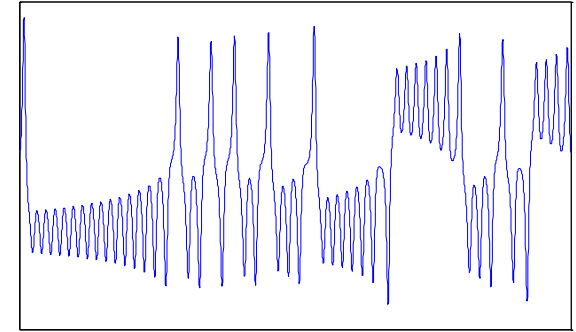
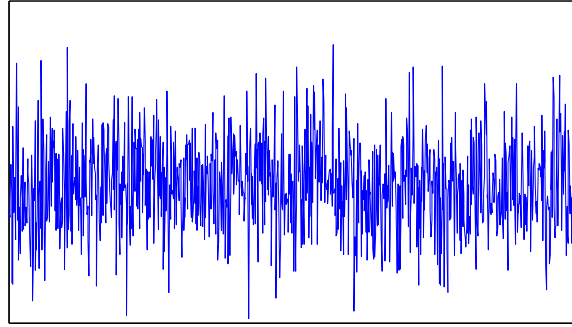
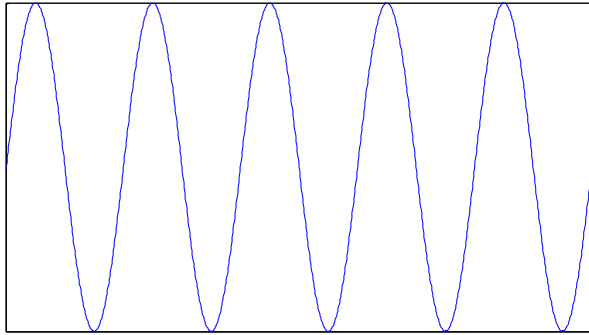
Recurrence Plot (Eckmann et al., 1987)

$$R_{i,j} = \begin{cases} 1: \vec{x}_i \approx \vec{x}_j \\ 0: \vec{x}_i \neq \vec{x}_j \end{cases} \quad i, j = 1, \dots, N$$

$$R_{i,j}(\varepsilon) = \Theta(\varepsilon - \|\vec{x}_i - \vec{x}_j\|) \quad i, j = 1, \dots, N$$



RPs of Different Systems



Periodic System

Arbitrary System

Lorenz System



Recurrence Quantification Analysis (RQA)

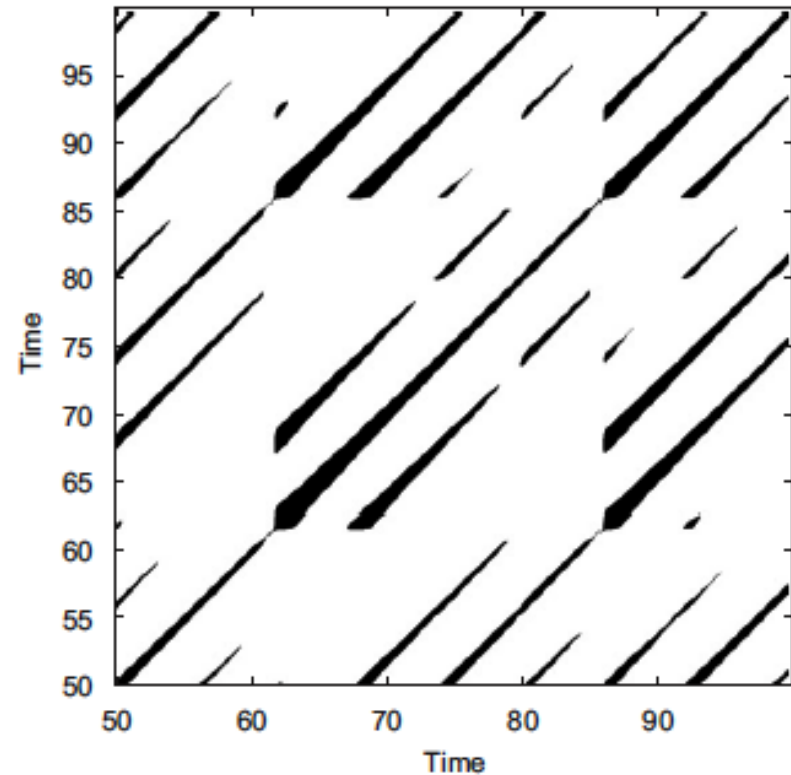
Quantifying the small scale structures for measuring the complexity of RPs

$$ENT = - \sum_{l=l_{min}}^N p(l) \ln p(l)$$

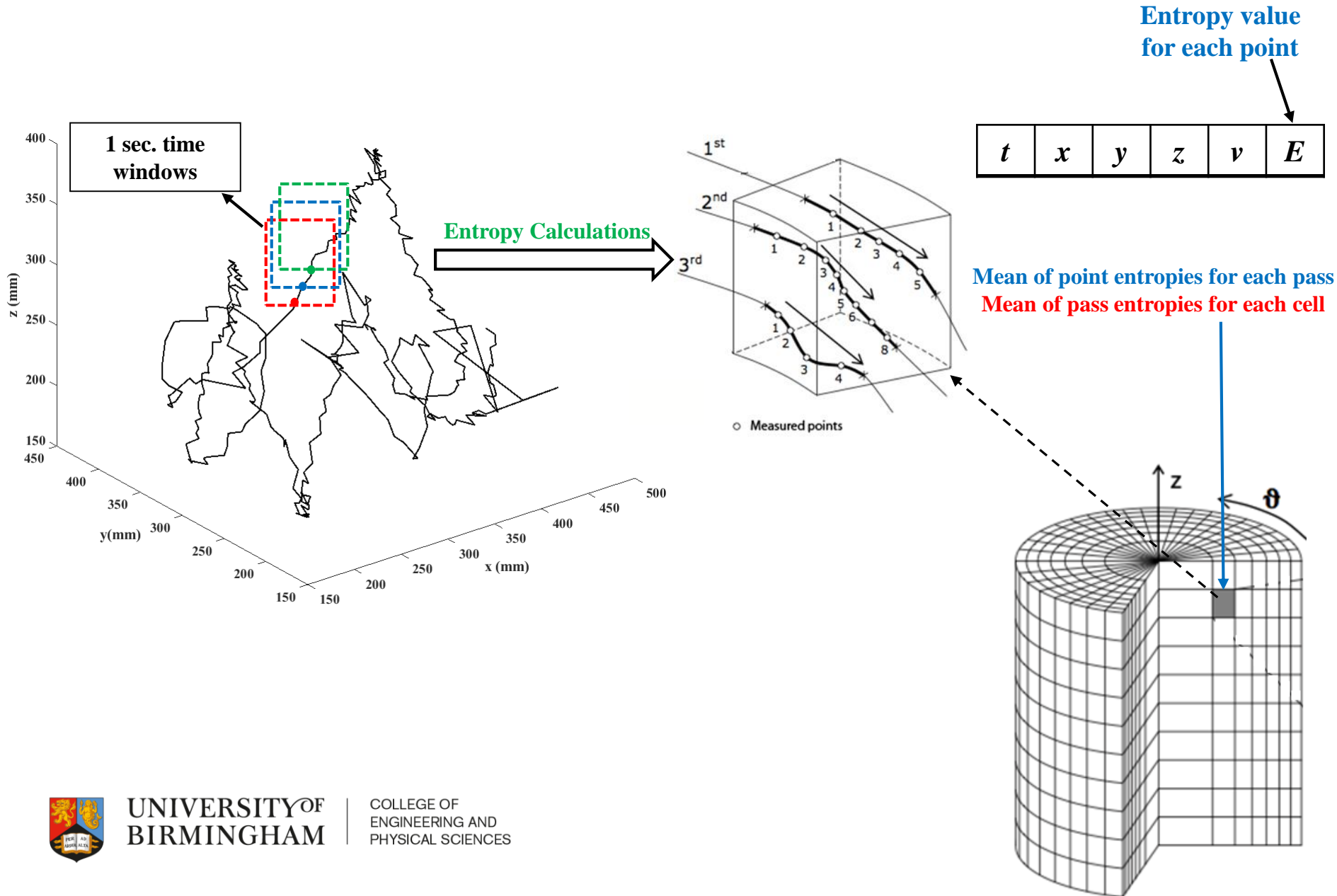
$$p(l) = \frac{P(l)}{N_l}$$

N_l = number of diagonal lines

$$l_{min} = 2$$

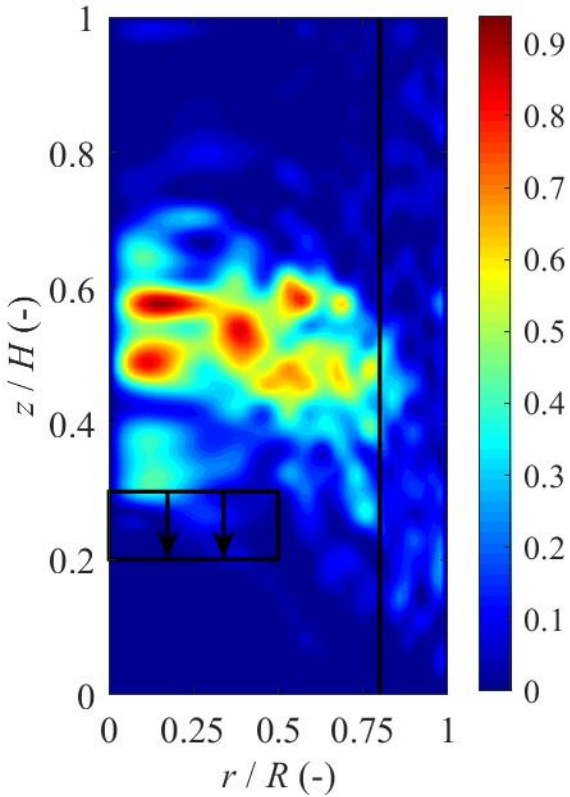


Entropy Calculation for Each Data Point

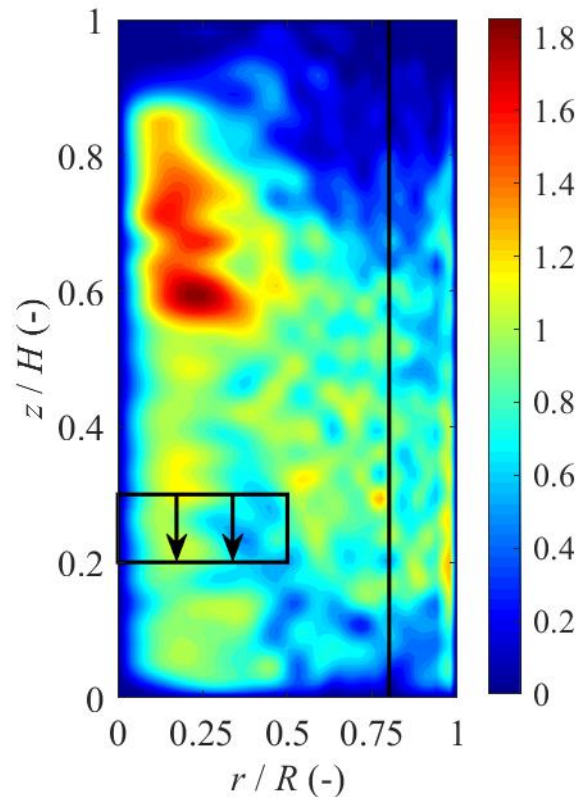


Azimuthally-Averaged Radial-Axial Maps of Entropy

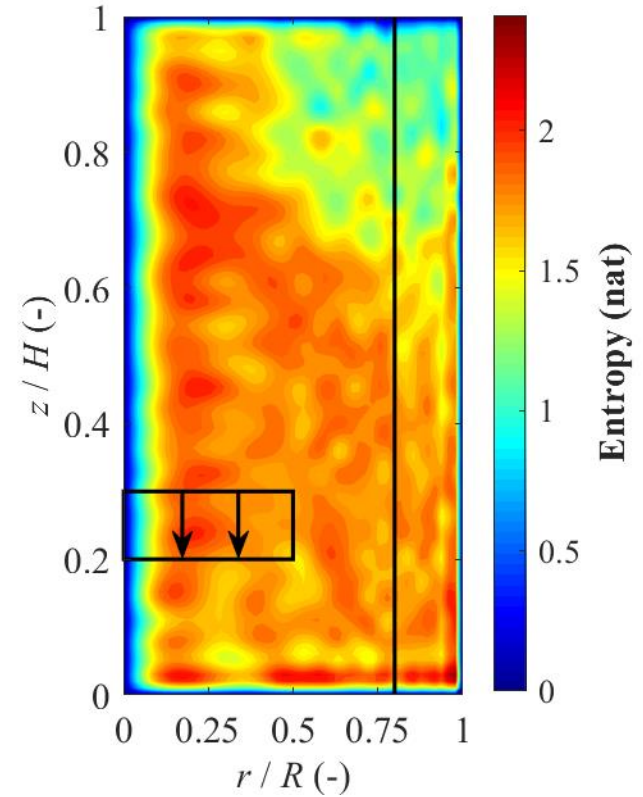
100 rpm



300 rpm



500 rpm



Pitch Blade Turbine Impeller
Down Pumping Mode, (PBT-D)

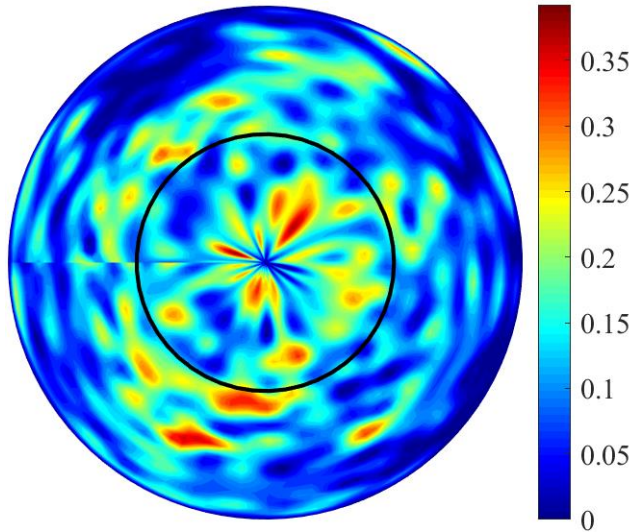


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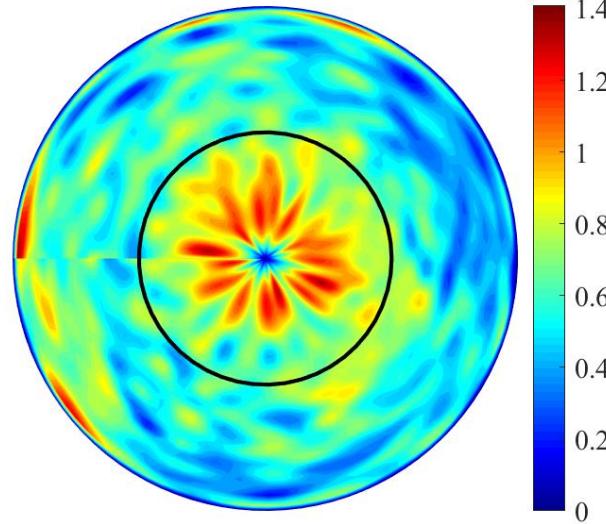
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Axially-Averaged Radial-Azimuthal Maps of Entropy

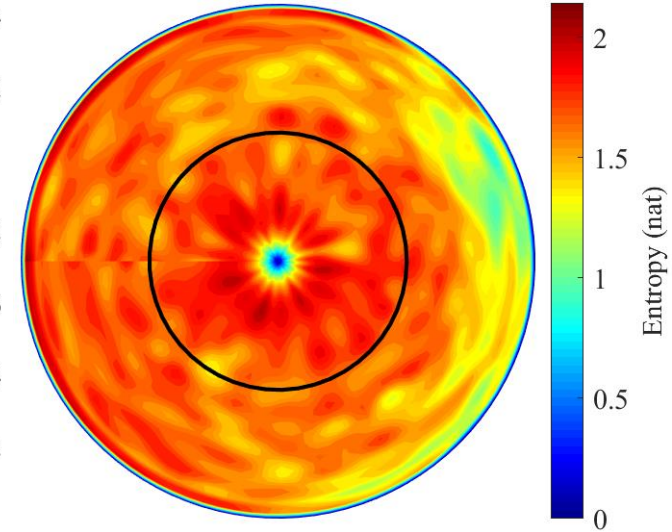
100 rpm



300 rpm



500 rpm



**Pitch Blade Turbine Impeller
Down Pumping Mode, (PBT-D)**

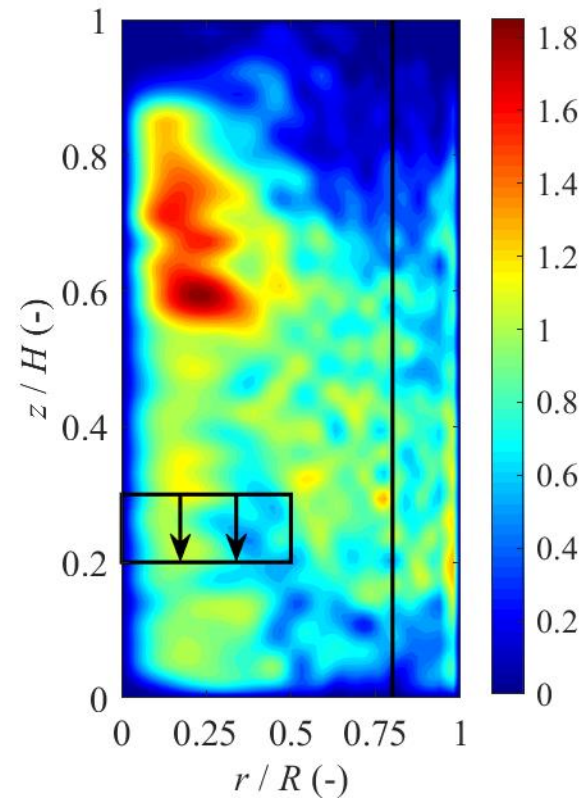


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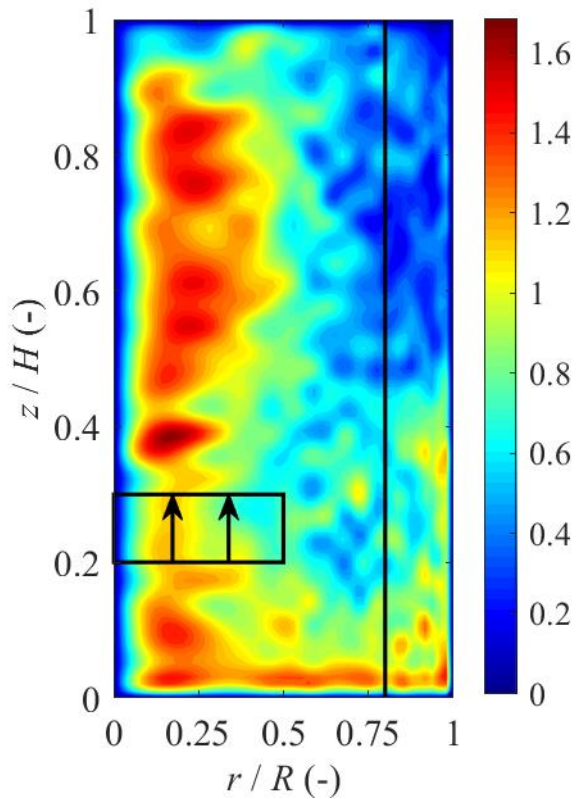
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Entropy Maps of Different Impellers

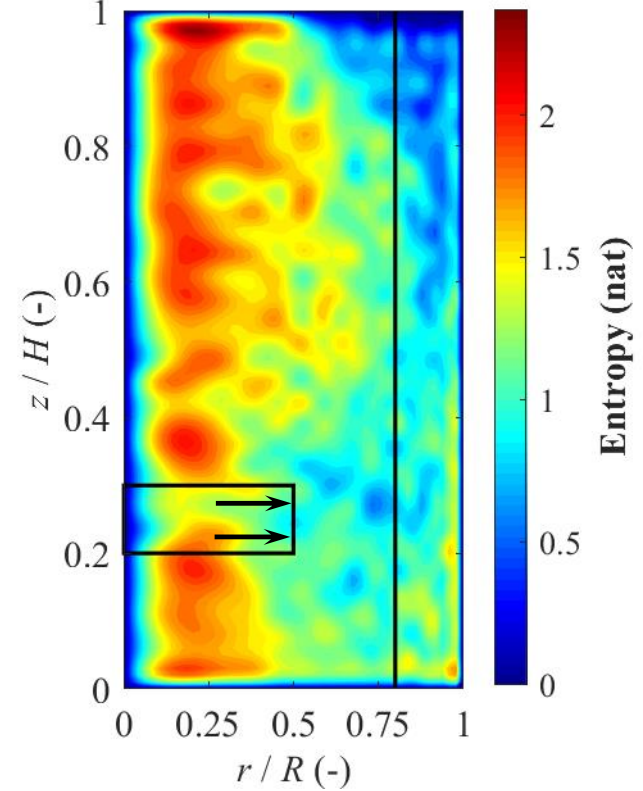
PBT-Down



PBT-Up



Rushton Disk Turbine (RTD)



300 rpm

F
A

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300 rpm

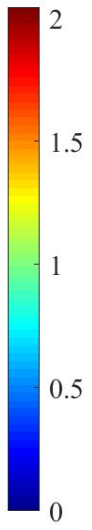
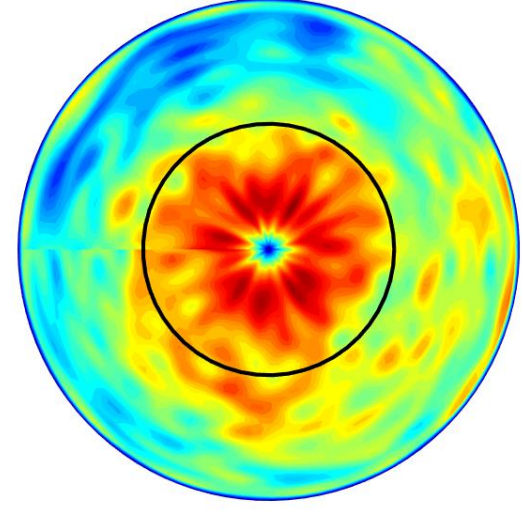
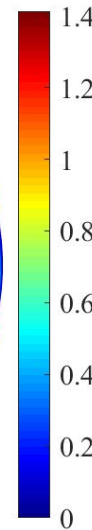
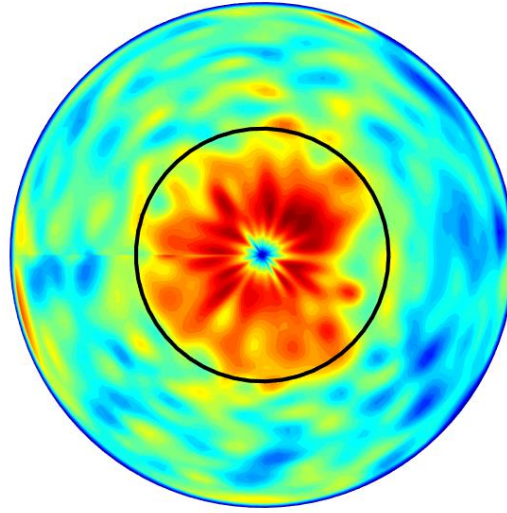
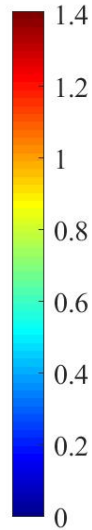
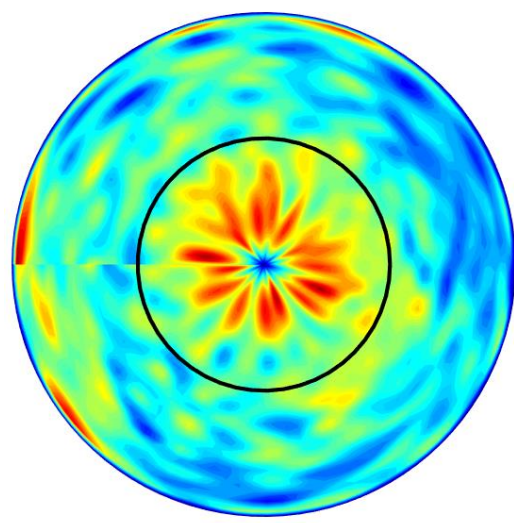


Entropy Maps of Different Impellers

PBT-Down

PBT-Up

**Rushton Disk Turbine
(RTD)**



300 rpm

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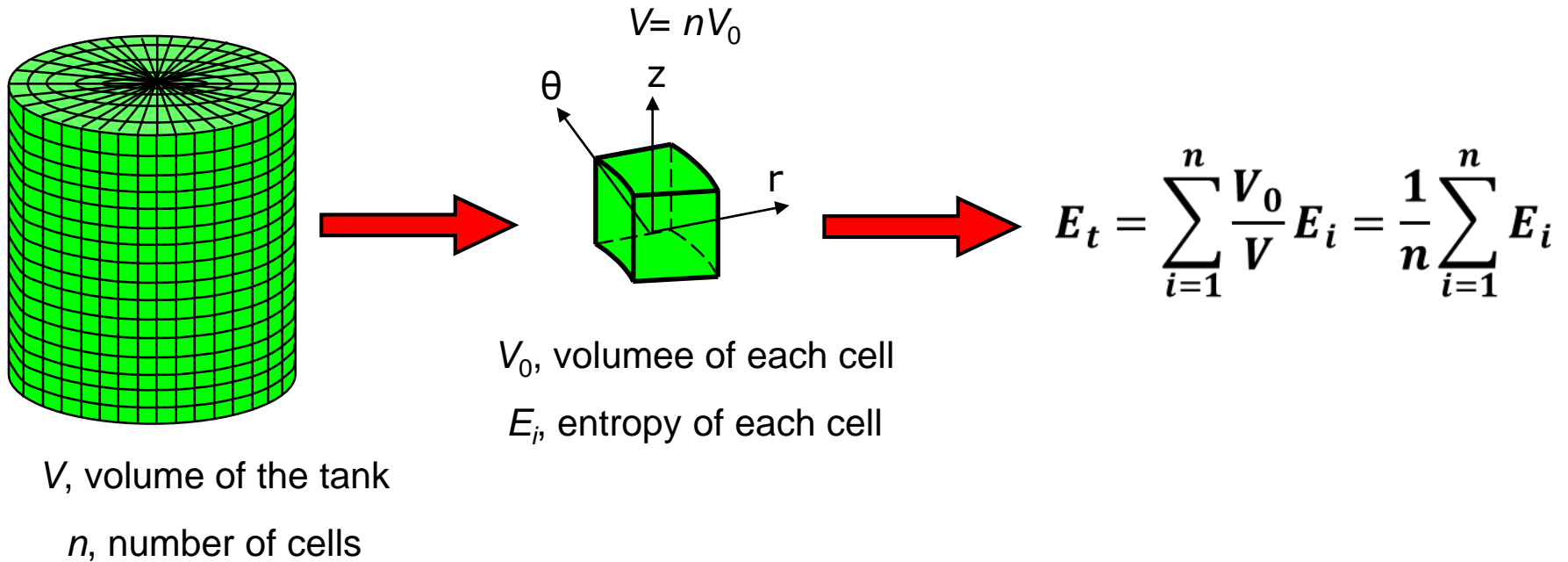
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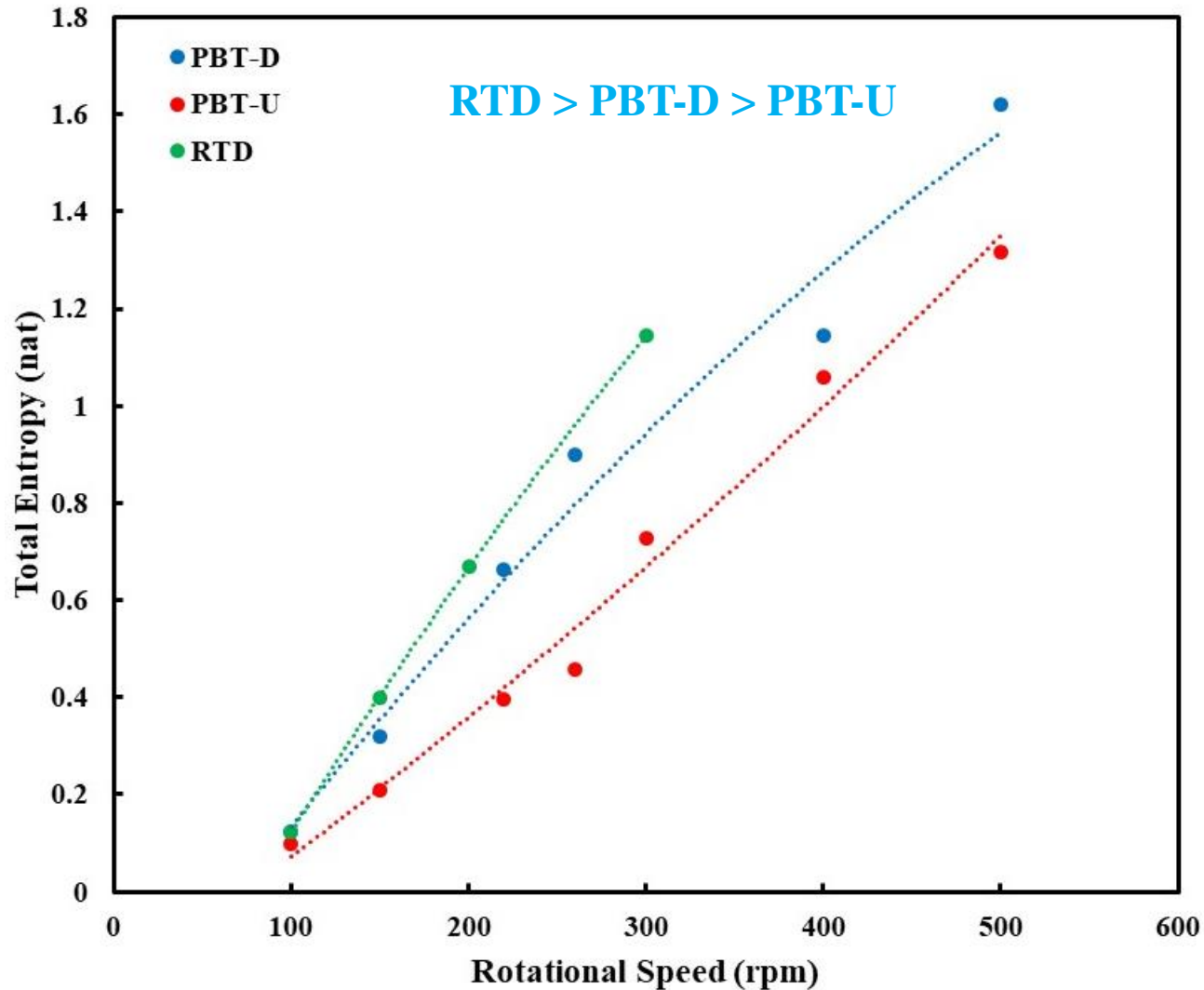
300 rpm



Global Entropy



Global Entropies, Single Phase Flow



Conclusions

- A new methodology based on the windowing recurrence quantification analysis has been developed for studying the local and global mixing performance in a batch stirred vessel.
- Global mixing index increases by increase in rotational speed of impeller.
- Global mixing index shows the order of the mixing rate under a constant impeller rotational speed becomes: $RTD > PBT-D > PBT-U$.
- Detailed information is obtained on global as well as local mixedness, allowing the identification of well-mixed and poorly-mixed cells.



Thanks for your attention



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